Math 274 | Quiz 10 | Name: ______________

1 Dec 2017 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{10}$ Find a particular solution of the equation $\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{f}(t)$ where

$$\mathbf{A} = \begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix}, \text{ and } \mathbf{f}(t) = \begin{bmatrix} 16e^{-t} \\ 12 \end{bmatrix}.$$

You will find the following helpful.

- The variation of parameters formula is $\mathbf{x}_p(t) = \mathbf{X}(t) \int \mathbf{X}^{-1}(t)\mathbf{f}(t)\,dt$.
- $\mathbf{X}(t) = \begin{bmatrix} e^{-t} & e^{3t} \\ 5e^{-t} & e^{3t} \end{bmatrix}$ is a fundamental matrix for $\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t)$.
- If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible then $\mathbf{A}^{-1} = \frac{1}{|\mathbf{A}|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$$\underline{X}^{-1} = \frac{1}{-4e^{2t}} \begin{bmatrix} e^{3t} & -e^{3t} \\ -5e^{-t} & e^{-t} \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -e^{t} & e^{t} \\ +5e^{-3t} & -e^{-3t} \end{bmatrix}$$

$$\underline{X}^{-1}\vec{f} = \frac{1}{4} \begin{bmatrix} -e^{t} & e^{t} \\ 5e^{-3t} & -e^{-3t} \end{bmatrix} \begin{bmatrix} 16e^{-t} \\ 12 \end{bmatrix} = \begin{bmatrix} -4 + 3e^{t} \\ 20e^{-4t} - 3e^{-3t} \end{bmatrix}$$

$$\int \underline{X}^{-1}\vec{f} = \begin{bmatrix} -4t + 3e^{t} \\ -5e^{-4t} + e^{-3t} \end{bmatrix}$$

$$\underline{X} \int \underline{X}^{-1}\vec{f} = \begin{bmatrix} e^{-t} & e^{3t} \\ 5e^{-t} & e^{3t} \end{bmatrix} \begin{bmatrix} -4t + 3e^{t} \\ -5e^{-4t} + e^{-3t} \end{bmatrix}$$

$$= \begin{bmatrix} -4te^{-t} + 3 - 5e^{-t} + 1 \\ -20te^{-t} + 15 - 5e^{-t} + 1 \end{bmatrix} = te^{-t} \begin{bmatrix} -4 \\ -20 \end{bmatrix} + e^{-t} \begin{bmatrix} -5 \\ -5 \end{bmatrix} + \begin{bmatrix} 4 \\ 16 \end{bmatrix}$$