Math 274 | Quiz 2 | Name: ________________
8 Sept 2017 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{4}$ Find and implicit solution to the initial value problem

$$\frac{dx}{dt} = (x^2+1)t, \qquad x(0) = 1.$$

$$\int \frac{dx}{x^2+1} = \int t\,dt$$

$$\arctan x = \frac{t^2}{2} + C$$

Substitute $t=0$, $x=1$

$$\arctan 1 = \frac{\pi}{4} = \frac{0^2}{2} + C \qquad \text{so} \qquad C = \frac{\pi}{4}$$

The solution is

$$\arctan x = \frac{t^2}{2} + \frac{\pi}{4}$$

[Continued on the other side.]

2. $\boxed{4}$ Find an explicit general solution to

$$\frac{dy}{dx} - \frac{2y}{x} = x^3.$$

$$\mu(x) = e^{\int -\frac{2}{x}\,dx} = e^{-2\ln|x|} = |x^{-2}| = x^{-2}$$

so

$$y = x^2 \int x \, dx$$

$$= x^2\left(\frac{x^2}{2} + C\right)$$

3. $\boxed{2}$ Last time we discussed the following theorem,

> **Theorem** If $P(x)$ and $Q(x)$ are continuous on (a,b) with $x_0 \in (a,b)$, then for any y_0, there exists a unique solution $y(x)$ on (a,b) that solves
>
> $$\frac{dy}{dx} + P(x)y = Q(x), \qquad y(x_0) = y_0.$$

Find the largest domain implied by the theorem that a unique solution exists for the equation

$$\frac{dy}{dx} + \frac{y}{x^2-1} = \sin\left(x^{2017}\right)$$

with each of the following initial coniditions. **Do not solve the equation.**

(a) $y(0) = 0$

$x \in (-1, 1)$

(b) $y(\pi) = e^2$

$x \in (1, \infty)$