Math 274 | Quiz 3 | Name: ______________

15 Sept 2017 | Show Appropriate Work | Scaled to $\boxed{10}$ points.

1. Consider the equation

$$\left(x - \frac{1}{x^2y}\right) dx + \left(y - \frac{1}{xy^2}\right) dy = 0.$$

(a) $\boxed{2}$ Show the equation is exact.

$$\frac{\partial}{\partial y}\left(x - \frac{1}{x^2y}\right) = \frac{1}{x^2y^2} = \frac{\partial}{\partial x}\left(y - \frac{1}{xy^2}\right)$$

(b) $\boxed{4}$ Find an implicit general solution.

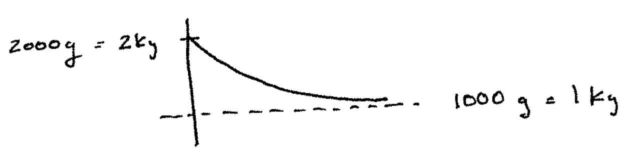
$$\frac{x^2}{2} + \frac{1}{xy} + \frac{y^2}{2} = C$$

(c) $\boxed{1}$ Find the solution satisfying $(x, y) = \cancel{(0,0)}$ $(1,1)$.

$$\frac{x^2}{2} + \frac{1}{xy} + \frac{y^2}{2} = 2$$

2. A 100 L tank is intially filled with a sugar water solution with concentration 20 g/L. A solution of concentration 10 g/L flows into the tank at 5 L/min. The tank is well mixed and the resulting mixture flows out at 5 L/min.

(a) $\boxed{2}$ Sketch a graph of the amount of sugar in the tank (in g) as a function of time.



(b) $\boxed{3}$ Write an initial value problem, i.e. a differential equation with initial data, to model the amount of sugar in the tank. **Do not solve.**

$$\frac{dx}{dt} = 5(10) - 5\left(\frac{x}{100}\right), \quad x(0) = 2000$$

$$= 50 - \frac{x}{20}$$

[Continued on reverse.]

- **Homogeneous Substitution.**
  If the equation is of the form $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$, use the substitution $v = \frac{y}{x}$.
- **Linear Combination Substitution.**
  If the equation is of the form $\frac{dy}{dx} = G(ax + by + c)$, use the substitution $v = ax + by + c$.
- **Bernoulli Substitution.**
  If the equation is of the form $\frac{dy}{dx} + P(x)y = Q(x)y^n$, use the substitution $v = y^{1-n}$.

3. $\boxed{8}$ Using an appropriate substitution find an explicit general solution for the equation

$$\frac{dx}{dt} = x + \frac{t}{x^2}.$$

$$\frac{dx}{dt} - x = \frac{t}{x^2}$$

$$x^2 \frac{dx}{dt} - x^3 = t \qquad \text{Let } v = x^3$$

$$\frac{dv}{dt} = 3x^2 \frac{dx}{dt}$$

$$\frac{dv}{dt} - 3v = 3t$$

$$\mu(t) = e^{-3t}$$

$$V = e^{3t} \int 3t\, e^{-3t}\, dt \qquad u = 3t \quad dv = e^{-3t}dt$$

$$du = 3\,dt \quad v = -\tfrac{1}{3} e^{-3t}$$

$$= e^{3t}\left[-te^{-3t} + \int e^{-3t}\, dt \right]$$

$$x^3 = e^{3t}\left(-te^{-3t} - \tfrac{1}{3} e^{-3t} + C \right)$$

$$x = \left[Ce^{3t} - \tfrac{1}{3} - t \right]^{1/3}$$