Math 274 | Quiz | Name: ____________
3 Oct 2017 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

Method of Undetermined Coefficients

To find a particular solution to

$$ay'' + by' + cy = P_m(t)e^{rt}$$

where $P_m(t)$ is a polynomial of degree m, use the form

$$y_p(t) = t^s \left(A_m t^m + \cdots + A_1 t + A_0\right) e^{rt};$$

if r is not a root of the associated auxiliary equation, take $s = 0$; if r is a simple root, take $s = 1$; and if r is a double root, take $s = 2$.

To find a particular solution to

$$ay'' + by' + cy = P_m(t)e^{\alpha t}\cos\beta t + Q_n(t)e^{\alpha t}\sin\beta t$$

where $P_m(t)$ and $Q_n(t)$ are polynomials of degree m and n, respectively, use the form

$$y_p(t) = t^s\left(A_k t^k + \cdots + A_1 t + A_0\right)e^{\alpha t}\cos\beta t + t^s\left(B_k t^k + \cdots + B_1 t + B_0\right)e^{\alpha t}\sin\beta t;$$

where k is the larger of m and n. If $\alpha + i\beta$ is not a root of the associated auxiliary equation, take $s = 0$; if so take $s = 1$.

1. For each of the following, specify the form of a particular solution suggested by the Method of Undetermined Coefficients. **DO NOT SOLVE FOR THE CONSTANTS.**

   (a) $\boxed{2}$ $\quad y'' + 9y = 2t + 8\sin 3t$

   $r^2 + 9 = 0$

   $r = \pm 3i$ $\qquad y_p = (At + B) + Ct\cos 3t + Dt\sin 3t$

   (b) $\boxed{2}$ $\quad y'' - 2y' + y = 2te^t - t^2e^{2t}$

   $r^2 - 2r + 1 = 0$

   $(r-1)^2 = 0$ $\qquad y_p = t^2(At + B)e^t + (Ct^2 + Dt + E)e^{2t}$

   $r = 1$, double

[Continued on reverse.]

2. $\boxed{6}$ Find a general solution for

$$y'' + 4y = 10te^t.$$

$r^2 + 4 = 0$

so $y_h = C_1 \cos 2t + C_2 \sin 2t$ solves the homogeneous

$y_p = (At + B)e^t$

$y_p' = (A + At + B)e^t$

$y_p'' = (2A + At + B)e^t$

$y_p'' + 4y_p = (At + 2A + B)e^t + (4At + 4B)e^t = 10te^t$

te^t: $5A = 10$ so $A = 2$

e^t: $2A + 5B = 0$ so $B = -\frac{4}{5}$

$y_p = (2t - \frac{4}{5})e^t$ is a particular solution

A general solution is then

$y = C_1 \cos 2t + C_2 \sin 2t + (2t - \frac{4}{5})e^t$