Math 274 | Quiz 8 | Name: ________________
27 Oct 2017 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{3}$ Express the following function using step functions and determine its Laplace transforms.

$$f(t) = \begin{cases} t, & t < 1 \\ e^{3t}, & 1 < t < 4 \\ 2, & 4 < t \end{cases}$$

$$= t + u(t-1)(e^{3t} - t) + u(t-4)(2 - e^{3t})$$

so $F(s) = \frac{1}{s^2} + e^{-s}\mathcal{L}\{e^{3t+3} - (t+1)\} + e^{-4s}\mathcal{L}\{2 - e^{3(t+4)}\}$

$$= \frac{1}{s^2} + e^{-s}\left[\frac{e^3}{s-3} - \frac{1}{s^2} - \frac{1}{s}\right] + e^{-4s}\left[\frac{2}{s} - \frac{e^{12}}{s-3}\right]$$

2. $\boxed{7}$ Applying the Laplace transform to the initial value problem

$$y'' + 2y' + y = \begin{cases} 0, & t < 2 \\ 4e^t, & 2 < t \end{cases}, \qquad y(0) = 1, y'(0) = -1$$

gives

$$Y(s) = \frac{s+1}{s^2+2s+1} + \frac{4e^{2-2s}}{(s-1)(s^2+2s+1)}.$$

Determine $y(t) = \mathscr{L}^{-1}\{Y(s)\}$, the solution to the given initial value problem.

$$\frac{s+1}{s^2+2s+1} = \frac{s+1}{(s+1)^2} = \frac{1}{s+1}$$

and

$$\frac{4}{(s-1)(s+1)^2} = \frac{A}{s-1} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$$

$$4 = A(s+1)^2 + B(s-1)(s+1) + C(s-1)$$

Let $s = 1$, $4 = 4A \Rightarrow A = 1$

Let $s = -1$, $4 = -2C \Rightarrow C = -2$

<u>Eq. coeff:</u> s^2: $0 = A + B \Rightarrow B = -1$

so

$$Y(s) = \frac{1}{s+1} + e^2 \cdot e^{-2s}\left[\frac{1}{s-1} - \frac{1}{s+1} - \frac{2}{(s+1)^2}\right].$$

finally,

$$y(t) = e^{-t} + e^2 u(t-2)\left[e^{t-2} - e^{-(t-2)} - 2(t-2)e^{-(t-2)}\right]$$