Math 274 | Quiz 9 | Name: ______________

17 Nov 2017 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

Consider

$$\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}.$$

1. $\boxed{7}$ Find the eigenvalues and the corresponding eigenvectors for $\mathbf{A}$.

$A - rI = \begin{bmatrix} 1-r & 3 \\ 2 & 2-r \end{bmatrix}$ so $\det(A - rI) = (1-r)(2-r) - 6$.

The Characteristic Equation is then $r^2 - 3r - 4 = 0$.

$(r-4)(r+1) = 0$.

The eigenvalues are $r_1 = 4$, $r_2 = -1$.

$r_1 = 4$: $\begin{bmatrix} -3 & 3 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, choose $\vec{u}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ as a corresponding eigenvector.

$r_2 = -1$: $\begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, choose $\vec{u}_2 = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ —— " ——.

2. $\boxed{3}$ Find a general solution to $\mathbf{x}' = \mathbf{A}\mathbf{x}$.

$$\vec{x} = c_1 e^{4t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 e^{-t}\begin{bmatrix} 3 \\ -2 \end{bmatrix}$$