Math 274 Exam 2 **Section:** _____ **Name:** _____

11 Oct 2018

Point values in boxes.

1. Find a general solution for the following.

(a) [4] $y'' - 3y' + 2y = 0$

$(\lambda - 1)(\lambda - 2) = 0$

$y = C_1 e^{t} + C_2 e^{2t}$

(b) [4] $y'' - 6y' + 9y = 0$

$(\lambda - 3)^2 = 0$

$y = C_1 e^{3t} + C_2 t e^{3t}$

(c) [4] $y'' - 6y' + 13y = 0$

$(\lambda - 3)^2 = -4 \qquad \lambda = 3 \pm 2i$

$y = C_1 e^{3t} \cos 2t + C_2 e^{3t} \sin 2t$

2. [4] Evaluate.

$$\operatorname{Im}\left(\frac{e^{3ti}}{2+i}\right)$$

$$= \operatorname{Im}\left[\frac{2-i}{5}(\cos 3t + i \sin 3t)\right]$$

$$= \frac{2}{5}\sin 3t - \frac{1}{5}\cos 3t$$

3. Consider the equation

$$y'' + 9y = f(t) \tag{1}$$

(a) [4] Find the general solution of the homogeneous equation associated with (1), i.e.,

$$y'' + 9y = 0.$$

$y = C_1 \cos 3t + C_2 \sin 3t$

(b) Find the form of a particular solution implied by the real variable Method of Undetermined Coefficients for (1) for the following forcing functions. For each, determine if the particular solution is bounded or unbounded for $t \geq 0$. Circle one (or more). **Do not solve for the undetermined coefficients.**

i. [4] $y'' + 9y = 3t$

Form $y_p =$ $At + B$

y_p is bounded / (unbounded) for $t \geq 0$.

ii. [4] $y'' + 9y = 2te^{-4t}$

Form $y_p =$ $(At + B)e^{-4t}$

y_p is (bounded) / unbounded for $t \geq 0$.

iii. [4] $y'' + 9y = \cos 3t$

Form $y_p =$ $At \cos 3t + Bt \sin 3t$

y_p is bounded / (unbounded for) $t \geq 0$.

iv. [4] $y'' + 9y = e^{-t} \sin 3t + e^{-t}$

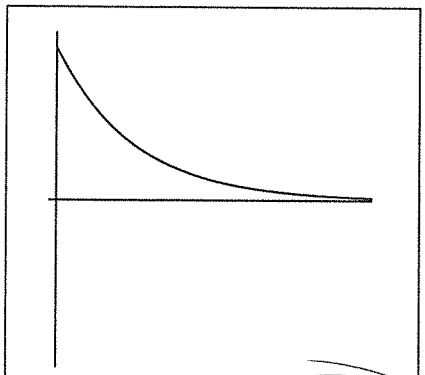
Form $y_p =$ $Ae^{-t} \sin 3t + Be^{-t} \cos 3t + Ce^{-t}$

y_p (is bounded) / unbounded for $t \geq 0$.

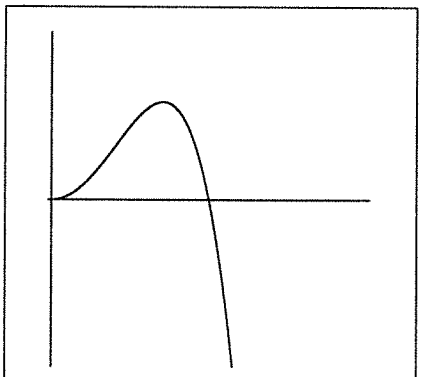
4. For $m, k > 0, \mu \geq 0$, the standard Mass-Spring system is given by $mx'' + \mu x' + kx = 0$.

(a) [1] Qualitatively, the behavior of a solution to a **Critically Damped system** is similar to an **Underdamped** / **Overdamped** system. (Circle One.)

(b) [9] The figures below show possible solution curves to this system. Label the system as **Undamped**, **Under**damped, **Over**damped, and/or **Not** a system of this form.



- Undamped
- Over
- Under
- Not



- Undamped
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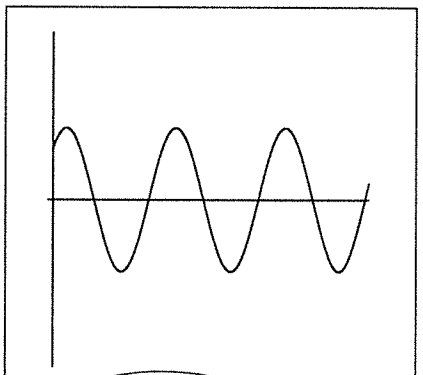

- Undamped
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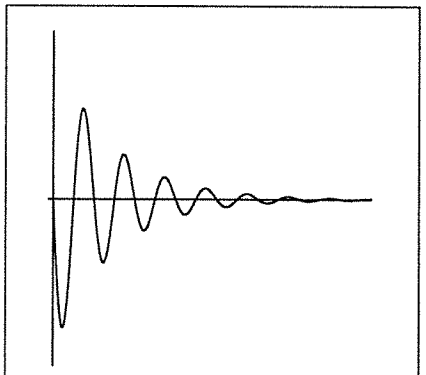

- Undamped
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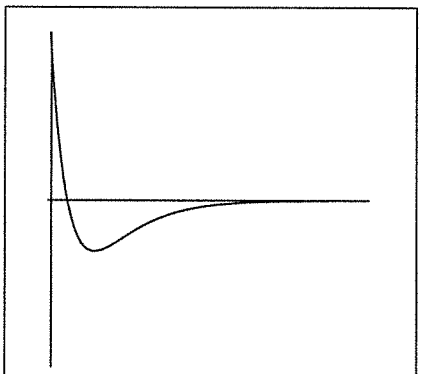

- Undamped
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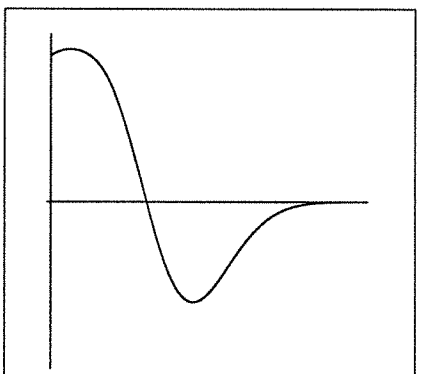

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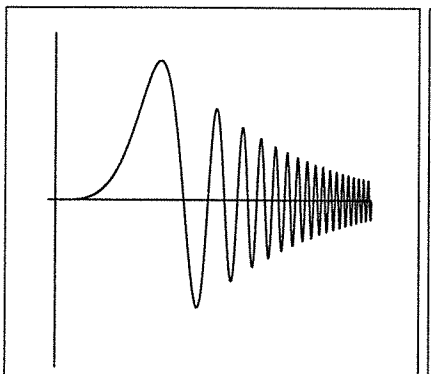

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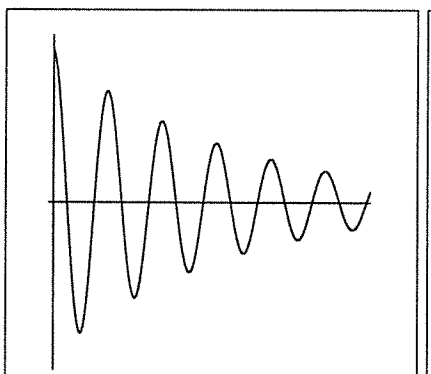

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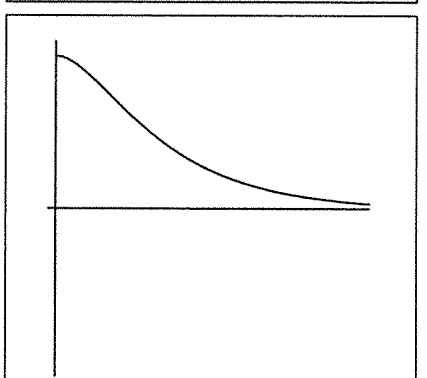

- Undamped
- Over
- Under
- Not

5. [16] Find a general solution

$$y'' + 3y' + 2y = e^{-t}\sin t.$$

$$(\lambda+1)(\lambda+2) = 0$$

$$y = C_1 e^{-t} + C_2 e^{-2t}$$

$$e^{-t}\sin t = \mathrm{Im}\left(e^{-t}e^{it}\right)$$

$$= \mathrm{Im}\left(e^{(-1+i)t}\right)$$

try $z_p = Ae^{(-1+i)t}$

$$z_p' = (-1+i)Ae^{(-1+i)t}$$

$$z_p'' = (-1+i)^2 Ae^{(-1+i)t} = -2iAe^{(-1+i)t}$$

Substituting gives

$$-2iAe^{(-1+i)t} + (-3+3i)Ae^{(-1+i)t} + 2Ae^{(-1+i)t} = e^{(-1+i)t}$$

so $A(i-1) = 1$

$$A = \frac{1}{i-1}\cdot\frac{i+1}{i+1} = \frac{1+i}{-2}$$

$$y_p = \mathrm{Im}\left[\left(-\tfrac{1}{2}-\tfrac{1}{2}i\right)e^{-t}e^{it}\right]$$

$$= -\tfrac{1}{2}e^{-t}\sin t - \tfrac{1}{2}e^{-t}\cos t$$

so $y = C_1e^{-t} + C_2e^{-2t} - \frac{1}{2}e^{-t}(\sin t - \cos t)$ is a general solution.

6. [6] The Bessel equation of order one-half

$$t^2y'' + ty' + \left(t^2 - \frac{1}{4}\right)y = 0, \quad t > 0$$

has solutions $y_1 = t^{-1/2}\cos t$ and $y_2 = t^{-1/2}\sin t$. Use the Wronskian to verify that y_1 and y_2 are linearly independent for $t > 0$.

$$W[y_1, y_2] = \det \begin{bmatrix} t^{-1/2}\cos t & t^{-1/2}\sin t \\ -\frac{1}{2}t^{-3/2}\cos t - t^{-1/2}\sin t & -\frac{1}{2}t^{-3/2}\sin t + t^{-1/2}\cos t \end{bmatrix}$$

$$= -\frac{1}{2}t^{-2}\sin t\cos t + t^{-1}\cos^2 t + \frac{1}{2}t^{-2}\sin t\cos t + t^{-1}\sin^2 t$$

$$= t^{-1} \neq 0$$

7. Consider the equation

$$t^2y'' - 2ty' - 4y = 0, \quad t > 0. \tag{2}$$

(a) [6] Show $y_1 = t^4$ is a solution to (2).

$$t^2(12t^2) - 2t(4t^3) - 4(t^4) = 0$$

(b) [8] Find a general solution to (2).

$$y'' - \frac{2}{t}y' - \frac{4}{t^2}y = 0$$

$$y_2 = t^4\int \frac{e^{-\int \frac{2}{t}dt}}{t^8}dt = t^4\int \frac{t^2}{t^8}dt = t^4 \cdot \frac{t^{-5}}{-5} = -\frac{1}{5t}$$

so $y = C_1t^4 + C_2 \cdot \frac{1}{t}$

8. [8] Find k and v_0 so that

$$y'' + ky = 0, \quad y(0) = 3, y'(0) = v_0$$

has solution with period π and amplitude 5.

Period π $\Rightarrow$ $k = 4$

$y = C_1 \cos 2t + C_2 \sin 2t$

Amplitude 5 $\Rightarrow$ $C_1^2 + C_2^2 = 25$

$y(0) = 3$ $\Rightarrow$ $C_1 = 3$ so $C_2 = \pm 4$

$2C_2 = V_0$ so $V_0 = \pm 8$

9. [8] A particular solution of

$$y'' + 9y = 9 \sin 3t \tag{3}$$

is given by

$$y_p = \frac{-3}{2} t \cos 3t.$$

Find the solution to (3) that satisfies the initial data $y(0) = 0, y'(0) = 0$.

$y = C_1 \cos 3t + C_2 \sin 3t - \frac{3}{2} t \cos 3t$ $\quad y(0) = 0 \Rightarrow C_1 = 0$

$y' = 3C_2 \cos 3t - \frac{3}{2} \cos 3t + \frac{9}{2} t \sin 3t$ $\quad y'(0) = 0 \Rightarrow C_2 = \frac{1}{2}$

$y = \frac{1}{2} \sin 3t - \frac{3}{2} t \cos 3t$

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Points							