Final Exam

$$\text{If } \mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ is invertible then } \mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Theorem 2.40.
Suppose that $\mathbf{A}$ is a 2×2 matrix with one eigenvalue λ of multiplicity 2, and suppose that the eigenspace of λ has dimension 1. Let $\mathbf{v}_1$ be a nonzero eigenvector, and choose $\mathbf{v}_2$ such that $(\mathbf{A} - \lambda \mathbf{I})\mathbf{v}_2 = \mathbf{v}_1$. Then

$$\mathbf{x}_1(t) = e^{\lambda t}\mathbf{v}_1 \text{ and}$$
$$\mathbf{x}_2(t) = e^{\lambda t}[\mathbf{v}_2 + t\mathbf{v}_1]$$

form a fundamental set of solutions to the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$.

Jacobian.
For a planar autonomous system given by

$$x' = f(x, y),$$
$$y' = g(x, y),$$

the Jacobian of $\begin{bmatrix} f \\ g \end{bmatrix}$ at $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$ is

$$J = J(x_0, y_0) = \begin{bmatrix} \frac{\partial f}{\partial x}(x_0, y_0) & \frac{\partial f}{\partial y}(x_0, y_0) \\ \frac{\partial g}{\partial x}(x_0, y_0) & \frac{\partial g}{\partial y}(x_0, y_0) \end{bmatrix}.$$

Poincaré Diagram: Classification of Phase Portaits in the $(\det A, \operatorname{Tr} A)$-plane