

Math 274 Inverse Laplace

Sections: 5.1-5.4

Due: 22 October 2018

Name: _____
Point values in .

1. [2] Using the provided table of Laplace transforms, show

$$\mathcal{L} \left\{ \frac{1}{2} (\sin 3t - 3t \cos 3t) \right\} = \frac{27}{(s^2 + 9)^2}.$$

$$\begin{aligned} & \int \left\{ \frac{1}{2} (\sin 3t - 3t \cos 3t) \right\} dt \\ &= \frac{1}{2} \left[\frac{3}{s^2 + 9} + 3 \frac{d}{ds} \left[\frac{s}{s^2 + 9} \right] \right] \\ &= \frac{1}{2} \left[\frac{3(s^2 + 9)}{(s^2 + 9)^2} + \frac{3(s^2 + 9 - 2s^2)}{(s^2 + 9)^2} \right] \\ &= \frac{\frac{27}{(s^2 + 9)^2}}{\frac{3s^2 + 27 - 3s^2 + 27}{(s^2 + 9)^2}} \end{aligned}$$

2. Determine the inverse Laplace transform of the following.

$$(a) [1] \quad G(s) = \frac{3}{2s+1} = \frac{3}{2} \cdot \frac{1}{s+\frac{1}{2}} \quad (d) [1] \quad K(s) = \frac{7}{(s-3)^6} = \frac{7}{5!} \cdot \frac{5!}{(s-3)^6}$$

$$g(t) = \frac{3}{2} e^{-t/2} \quad K(t) = \frac{7}{5!} t^5 e^{3t}$$

$$(b) [1] \quad H(s) = \frac{6}{4-s} = \frac{-6}{s-4} \quad (e) [1] \quad M(s) = \frac{2s+3}{s^2+2}$$

$$h(t) = -6e^{4t} \quad m(t) = 2 \cos(\sqrt{2}t) + \frac{3}{\sqrt{2}} \sin(\sqrt{2}t)$$

$$(c) [2] \quad N(s) = \frac{s}{s^2 + 6s + 8} \quad (f) [2] \quad N(s) = \frac{3s}{s^2 - 6s + 13}$$

$$\frac{s}{s^2 + 6s + 8} = \frac{A}{s+2} + \frac{B}{s+4} = \frac{3(s-3) + 9}{(s-3)^2 + 2^2}$$

$$s = A(s+4) + B(s+2)$$

$$s=-4 \Rightarrow B=2$$

$$s=-2 \Rightarrow A=-1$$

$$h(t) = 2e^{-4t} - e^{-2t}$$

$$h(t) = 3e^{3t} \cos 2t$$

$$+ \frac{9}{2} e^{3t} \sin 2t$$

3. [4] Apply the Laplace transform to the initial value problem

$$y'' + 3y' + 2y = 4e^{-2t}, \quad y(0) = 2, y'(0) = -7$$

to express $Y(s) = \mathcal{L}\{y(t)\}$ in the form $Y(s) = \frac{P(s)}{Q(s)}$; for example, (1) below is of this form. In particular, express $P(s)$ as a single polynomial, i.e. multiply out and combine like terms. Express $Q(s)$ factored into linear and/or irreducible quadratic terms.

Do not find the inverse Laplace transform.

$$s^2 Y - 2s + 7 + 3(sY - 2) + 2Y = \frac{4}{s+2}$$

$$\begin{aligned} Y(s^2 + 3s + 2) &= \frac{4}{s+2} + 2s - 1 \\ &= \frac{4 + (2s-1)(s+2)}{(s+2)} \end{aligned}$$

$$Y = \frac{2s^2 + 3s + 2}{(s+1)(s+2)^2}$$

4. [6] Applying the Laplace transform to the initial value problem

$$y'' + 2y' + 10y = -20e^{-2t}, \quad y(0) = 1, y'(0) = 7$$

gives the following

$$Y(s) = \frac{s^2 + 11s - 2}{(s+2)(s^2 + 2s + 10)}. \quad (1)$$

Determine $y(t) = \mathcal{L}^{-1}\{Y(s)\}$, the solution to the given initial value problem.

$$\frac{s^2 + 11s - 2}{(s+2)(s^2 + 2s + 10)} = \frac{A}{s+2} + \frac{Bs + C}{s^2 + 2s + 10}$$

$$Y(s) = \frac{-2}{s+2} + \frac{3s + 9}{(s+1)^2 + 3^2}$$

$$s^2 + 11s - 2 = A(s^2 + 2s + 10) + (Bs + C)(s+2) \quad = \frac{-2}{s+2} + \frac{3(s+1)}{(s+1)^2 + 3^2} + \frac{2 \cdot 3}{(s+1)^2 + 3^2}$$

$$s=-2 : -20 = A(10) \quad \text{so} \quad A = -2$$

$$s^2 : 1 = A + B \quad \text{so} \quad B = 3$$

$$s^0 : -2 = 10A + 2C \quad \text{so} \quad C = 9$$

$$y(t) = 3e^{-t} \cos 3t + 2e^{-t} \sin 3t - 2e^{-2t}$$