Math 274 MUCs Examples and Forms

Section: 4.4

It is important to practice the correct form for the Method of Undetermined Coefficients. Below you will find a number of examples and some exercises for you to try. Do not try to solve for the constants, just specify the form a solution should have.

1. When there is no overlap with solutions to the homogeneous equation we use a form based on the inhomogeneity.

   Examples:

   (a) $y'' - y' - 6y = 5$ $\qquad y_p = A$

   (b) $y'' - y' - 6y = 5t$ $\qquad y_p = At + B$

   (c) $y'' - y' - 6y = 5t^2$ $\qquad y_p = At^2 + Bt + C$

   (d) $y'' - y' - 6y = e^{5t}$ $\qquad y_p = Ae^{5t}$

   (e) $y'' - y' - 6y = (t+3)e^{5t}$ $\qquad y_p = (At + B)e^{5t}$

   (f) $y'' - y' - 6y = \cos 5t$ $\qquad y_p = A\cos 5t + B\sin 5t$

   (g) $y'' - y' - 6y = t\cos 5t$ $\qquad y_p = (At + B)\cos 5t + (Ct + D)\sin 5t$

   (h) $y'' - y' - 6y = e^{3t}\cos 5t$ $\qquad y_p = Ae^{3t}\cos 5t + Be^{3t}\sin 5t$

   (i) $y'' + 25y = e^{3t}\cos 5t$ $\qquad y_p = Ae^{3t}\cos 5t + Be^{3t}\sin 5t$

   Exercises:

   $\lambda_1 = 2,\ \lambda_2 = -1$

   (a) $y'' - y' - 2y = 4e^{3t}$ $\qquad y_p = Ae^{3t}$

   (b) $y'' - y' - 2y = -4t$ $\qquad y_p = At + B$

   (c) $y'' - y' - 2y = 4e^{3t} - 4t$ $\qquad y_p = Ae^{3t} + (Bt + C)$ combine

   (d) $y'' - y' - 2y = 20\cos 2t$ $\qquad y_p = A\cos 2t + B\sin 2t$

   $\lambda = \pm 5i$

   (e) $y'' + 25y = e^{5t}$ $\qquad y_p = Ae^{5t}$

   (f) $y'' + 25y = e^{t}\sin 5t$ $\qquad y_p = Ae^{t}\cos 5t + Be^{t}\sin 5t$

   (g) $y'' + 25y = t^2 - 7$ $\qquad y_p = At^2 + Bt + C$

   (h) $y'' + 25y = \sin t + \cos 2t$ $\qquad y_p = A\cos t + B\sin t + C\cos 2t + D\sin 2t$

   $\lambda = -1$, repeated

   (i) $y'' + 2y' + y = (3t+4)e^{2t}$ $\qquad y_p = (At + B)e^{2t}$

   (j) $y'' + 2y' + y = 7t + \cos t$ $\qquad y_p = At + B + C\cos t + D\sin t$

   (k) $y'' + 2y' + y = 7t\sin t$ $\qquad y_p = (At+B)\cos t + (Ct+D)\sin t$

   (l) $y'' + 2y' + y = 2te^{-t}\sin t$ $\qquad y_p = (At+B)e^{-t}\cos t + (Ct+D)e^{-t}\sin t$

2. When there is overlap with solutions to the homogeneous equation we supplement the form with an extra t, or t^2 in the case of repeated roots.

Examples:

(a) $y'' - y = e^t$ $\qquad y_p = Ate^t$

(b) $y'' - y = e^t + te^{3t}$ $\qquad y_p = Ate^t + (Bt + C)e^{3t}$

(c) $y'' - y = t^2e^t + 4$ $\qquad y_p = t(At^2 + Bt + C)e^t + D$

(d) $y'' + 9y = \cos 3t$ $\qquad y_p = At\cos 3t + Bt\sin 3t$

(e) $y'' + 9y = t\sin 3t$ $\qquad y_p = t(At + B)\cos 3t + t(Ct + D)\sin 3t$

(f) $y'' - 4y' + 4y = e^{2t}$ $\qquad y_p = At^2e^{2t}$

(g) $y'' - 4y' + 4y = (7t^2 + 3)e^{2t} + 3t - 1$ $\qquad y_p = t^2(At^2 + Bt + C)e^{2t} + (Dt + E)$

Exercises:

$\lambda_1 = 2$, $\lambda_2 = -1$

(a) $y'' - y' - 2y = 18e^{2t}$ $\qquad y_p = Ate^{2t}$

(b) $y'' - y' - 2y = te^{2t} + 4$ $\qquad y_p = t(At+B)e^{2t} + C$

$\lambda = \pm 2i$

(c) $y'' + 4y = \sin 2t$ $\qquad y_p = At\cos 2t + Bt\sin 2t$

(d) $y'' + 4y = t\sin 2t + 3\cos 2t$ $\qquad y_p = t(At+B)\cos 2t + t(Ct+D)\sin 2t$

$\lambda = -1$, repeated

(e) $y'' + 2y' + y = e^{-t} + t^2$ $\qquad y_p = At^2e^{-t} + (Bt^2 + Ct + D)$

(f) $y'' + 2y' + y = (t + 3)e^{-t}$ $\qquad y_p = t^2(At+B)e^{-t}$

3. It is often convenient to consider complex exponentials when using the Method of Undetermined Coefficients.

Examples:

(a) $y'' - y' - 6y = 5\sin 2t = \mathrm{Im}(5e^{2ti})$ $\qquad z_p = Ae^{2ti}$

(b) $y'' - y' - 6y = 5t\sin 2t = \mathrm{Im}(5te^{2ti})$ $\qquad z_p = (At + B)e^{2ti}$

(c) $y'' - y' - 6y = e^t\sin 2t = \mathrm{Im}(e^{(1+2i)t})$ $\qquad z_p = Ae^{(1+2i)t}$

(d) $y'' + 9y = \cos 3t = \mathrm{Re}(e^{3it})$ $\qquad z_p = Ate^{3it}$

Exercises:

(a) $y'' + 4y = \cos 2t = \mathrm{Re}(e^{2ti})$ $\qquad z_p = Ate^{2ti}$

(b) $y'' - y' - 2y = 20\sin 2t = \mathrm{Im}(20e^{2it})$ $\qquad z_p = Ae^{2ti}$

(c) $y'' - y' - 2y = 3t\cos 2t = \mathrm{Re}(3te^{2it})$ $\qquad z_p = (At+B)e^{2ti}$

(d) $y'' - y' - 2y = e^{3t}\sin 2t = \mathrm{Im}(e^{(3+2i)t})$ $\qquad z_p = Ae^{(3+2i)t}$