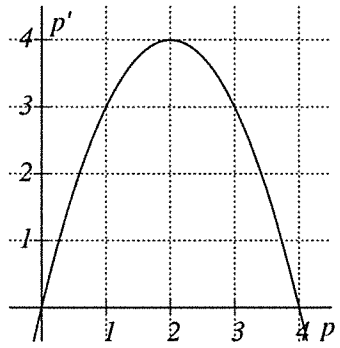
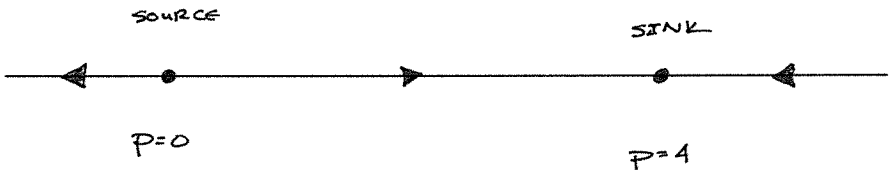
Math 274 **Phase Line**
Section: 2.9

1. Consider a population model of bison on the Great Plains given by $\frac{dp}{dt} = p(4-p)$ where p is measured in millions, see figure.



(a) Sketch the phase line for the model. Label each equilibrium as a sink (stable), source (unstable), or node (neither). (Sketch your phase line horizontally for convenience with $-\infty$ on the left and $+\infty$ on the right, as usual.)

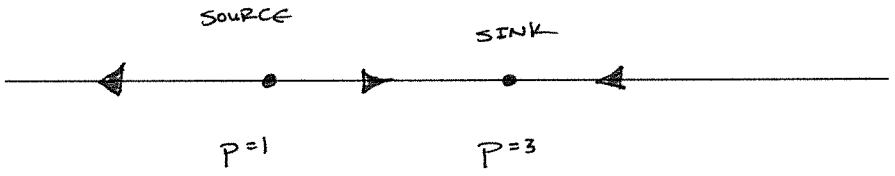


(b) What is the carrying capacity of the system, i.e. what is the asymptotic behavior of a population satisfying $p(0) > 0$ as $t \to \infty$.

$P \longrightarrow 4$ as $t \longrightarrow \infty$

2. If a constant harvesting rate is introduced into the bison population model from above we have $\frac{dp}{dt} = p(4-p) - 3$, where p is measured in millions.

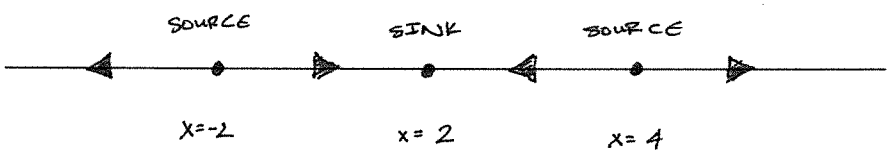
(a) Sketch the phase line for the model with harvesting. Label each equilibrium as a sink, source, or node.



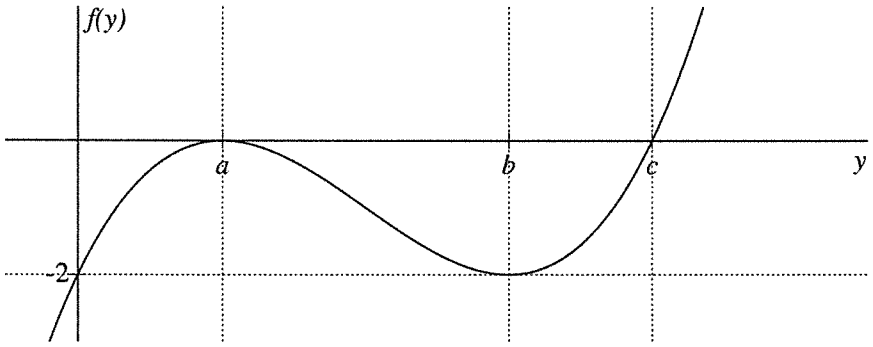
(b) What initial populations, i.e. $p(0) = p_0 > 0$, will lead to extinction with constant harvesting?

$0 < P_0 < 1$

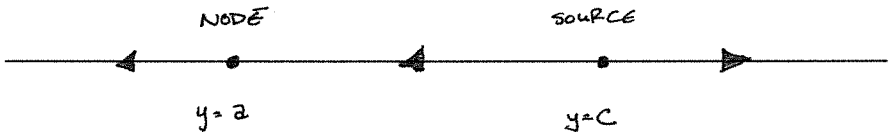
3. Sketch the phase line for $x' = (x+2)(x-2)(x-4)$. Label each equilibrium as a sink (stable), source (unstable), or node (neither).



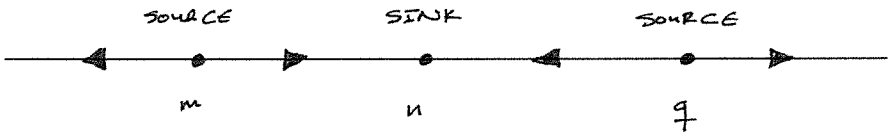
4. The graph of $f(y)$ is given below, use it to answer the following.



(a) Sketch the phase line for $y' = f(y)$. Identify each equilibria as a sink, source, or node.

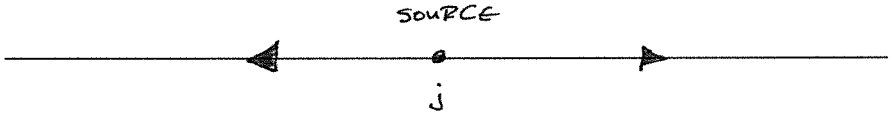


(b) For what values of k does $y' = f(y) + k$ have three equilibria? Sketch a representative phase line for one of those k values[1] including the standard arrows and labels.



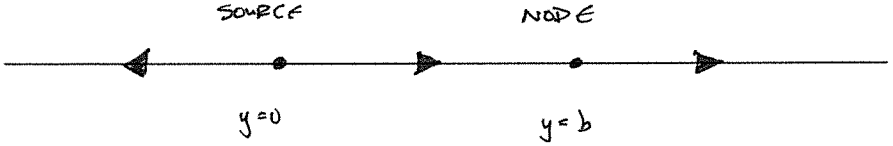
$0 < K < 2$

(c) For what values of k does $y' = f(y) + k$ have one equilibrium? Sketch a representative phase line for one of those k values[2] including the standard arrows and labels.



$K < 0$ OR $K > 2$

(d) For what values of $k \neq 0$ does $y' = f(y) + k$ have two equilibria? Sketch the phase line for that k value, including the standard arrows and label.



[1] Obviously we don't know the y values; use $m < n < q$ for labels.
[2] Obviously we don't know the y value; use j for the label.