

Math 274 Quiz 4

Sections: 4.1-4.3

28 September 2018

Name: _____

Point values in boxes.

1. For $t > 0$, show that $y_1 = t^3$ and $y_2 = t^{-1}$ form a fundamental solution set for

$$t^2 y'' - ty' - 3y = 0. \quad (1)$$

We will do so as follows.

- (a) 2 Show y_1 and y_2 are solutions of (1).

$$t^2 y_1'' - ty_1' - 3y_1 = t^2(6t) - t(3t^2) - 3(t^3) = 0$$

$$t^2 y_2'' - ty_2' - 3y_2 = t^2(2t^{-3}) - t(-t^{-2}) - 3(t^{-1}) = 0$$

- (b) 2 Use the Wronskian to show y_1 and y_2 are linearly independent.

$$W = \det \begin{bmatrix} t^3 & t^{-1} \\ 3t^2 & -t^{-2} \end{bmatrix} = -t - 3t = -4t \neq 0 \text{ for } t > 0.$$

2. 3 Find a general solution for each of the following.

(a) $y'' + 3y' + 2y = 0$

$$\lambda^2 + 3\lambda + 2 = 0$$

$$(\lambda+1)(\lambda+2) = 0$$

$$y = C_1 e^{-2t} + C_2 e^{-t}$$

(b) $y'' + 6y' + 9y = 0$

$$\lambda^2 + 6\lambda + 9 = 0$$

$$(\lambda+3)^2 = 0$$

$$y = C_1 e^{-3t} + C_2 t e^{-3t}$$

3. 3 Find the solution to the initial value problem

$$y'' - y' - 2y = 0, \quad y(0) = 5, y'(0) = 1.$$

$$\lambda^2 - \lambda - 2 = 0$$

$$(\lambda-2)(\lambda+1) = 0$$

$$y = C_1 e^{2t} + C_2 e^{-t}$$

$$y(0) = C_1 + C_2 = 5$$

$$y'(0) = 2C_1 - C_2 = 1$$

$$3C_1 = 6 \text{ so } C_1 = 2$$

$$C_2 = 3$$

$$y = 2e^{2t} + 3e^{-t}$$