Math 274 Quiz 5 **Name:** _____

Sections: 5.1-5.5 Point values in $\boxed{\text{boxes}}$.

26 October 2018

1. $\boxed{3}$ Express the following function using Heaviside functions and determine its Laplace transforms.

$$f(t) = \begin{cases} 1, & t < 2 \\ 2t, & 2 < t < 3 \\ e^{-t}, & 3 < t \end{cases}$$

$$= 1 + H(t-2)\left[2t - 1\right] + H(t-3)\left[e^{-t} - 2t\right]$$

$$F(s) = \frac{1}{s} + e^{-2s}\mathcal{L}\left\{2(t+2) - 1\right\} + e^{-3s}\mathcal{L}\left\{e^{-(t+3)} - 2(t+3)\right\}$$

$$= \frac{1}{s} + e^{-2s}\left(\frac{2}{s^2} + \frac{3}{s}\right) + e^{-3s}\left(e^{-3}\cdot\frac{1}{s+1} - \frac{2}{s^2} - \frac{6}{s}\right)$$

2. $\boxed{3}$ Find the inverse Laplace transform of the following.

$$F(s) = \frac{4se^{-2s}}{s^2 + 2s - 3} = e^{-2s}\left(\frac{1}{s-1} + \frac{3}{s+3}\right)$$

$$\frac{4s}{(s+3)(s-1)} = \frac{A}{s+3} + \frac{B}{s-1}$$

$$4s = A(s-1) + B(s+3)$$

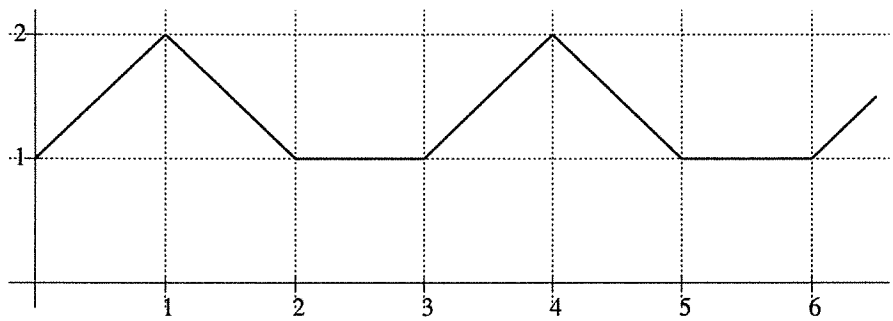
$$f(t) = H(t-2)\left[e^{t-2} + 3e^{-3(t-2)}\right]$$

$s=1$: $4 = 4B$ so $B = 1$

$s=-3$ $-12 = A(-4)$ so $A = 3$

CONTINUED ON REVERSE.

3. $\boxed{4}$ Compute the Laplace transform of the periodic function $f(t)$ given by the graph below.



$$f_3 = (1+t) + H(t-1)\left[3 - t - (1+t)\right] + H(t-2)\left[1 - (3-t)\right] - H(t-3)$$

$$= (1+t) + H(t-1)\left[2 - 2t\right] + H(t-2)\left[t-2\right] - H(t-3)$$

$$F_3 = \frac{1}{s} + \frac{1}{s^2} + e^{-s}\,\mathcal{L}\{2 - 2(t+1)\} + e^{-2s}\cdot\frac{1}{s^2} - e^{-3s}\cdot\frac{1}{s}$$

$$= \frac{1}{s} + \frac{1}{s^2} + e^{-s}\left(\frac{-2}{s^2}\right) + e^{-2s}\cdot\frac{1}{s^2} - e^{-3s}\cdot\frac{1}{s}$$

$$F = \frac{s + 1 - 2e^{-s} + e^{-2s} - se^{-3s}}{s^2\left(1 - e^{-3s}\right)}$$