Math 274 Quiz 7 **Name:** _____

Sections: 5.1-5.7 Point values in boxes.

2 November 2018

1. [2] The solution to the initial value problem

$$y'' + 2y' - 5y = 0, \qquad y(0) = 0, y'(0) = -6$$

is $y = -3e^{-t}\sin 2t$. Find the impulse required so that the system comes to rest the first time it returns to the equilibrium at $t = \pi/2$. Express your solution as a delta function.

$y' = 3e^{-t}\sin 2t - 6e^{-t}\cos 2t$

$y'(\frac{\pi}{2}) = 6e^{-\pi/2}$ so impulse is given by $-6e^{-\pi/2}\delta(t-\frac{\pi}{2})$

2. [2] Express the following convolution as an integral. **DO NOT EVALUATE.**

$$e^{2t} * \sin 3t = \int_0^t e^{2u}\sin\left(3(t-u)\right)du = \int_0^t e^{2(t-u)}\sin 3u\,du$$

3. [6] Assume $g(t)$ is piecewise continuous and of exponential order and consider the initial value problem

$$y' + y = g(t) + H(t-4), \qquad y(0) = 3.$$

Find the solution. Express your solution in terms of a convolution.

$sY - 3 + Y = G(s) + \frac{1}{s}e^{-4s}$

$\frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$

$Y = \frac{1}{s+1}\cdot G(s) + \frac{3}{s+1} + \frac{1}{s(s+1)}e^{-4s}$

$y = e^{-t} * g(t) + 3e^{-t} + H(t-4)\left[1 - e^{-(t-4)}\right]$