Math 274 | 4.2/4.3 Thing | Name: ____________
Due: 7 Feb 2018 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

Yesterday we showed that for

$$ay'' + by' + cy = 0 \tag{1}$$

the corresponding Characteristic Equation was

$$ar^2 + br + c = 0. \tag{2}$$

If the roots to (2) are distinct real $r_1 \neq r_2$, then a general solution to (1) is given by $y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$. If the roots to (2) are repeated real r, then a general solution to (1) is given by $y = c_1 e^{rt} + c_2 t e^{rt}$. And, if the roots to (2) are complex conjugates $r = \alpha \pm i\beta$ then a general solution to (1) is given by $y = c_1 e^{\alpha t}\cos\beta t + c_2 e^{\alpha t}\sin\beta t$.

1. Find a general solution for the following.

(a) $\boxed{1}$ $y'' - y' - 6y = 0$

$r_1 = 3$, $r_2 = -2$

$r^2 - r - 6 = 0$

$(r-3)(r+2) = 0$

$y = C_1 e^{3t} + C_2 e^{-2t}$

(b) $\boxed{1}$ $y'' + 6y' + 9y = 0$

$r = -3$

$r^2 + 6r + 9 = 0$

$(r+3)^2 = 0$

$y = C_1 e^{-3t} + C_2 t e^{-3t}$

(c) $\boxed{1}$ $y'' + 25y = 0$

$r = \pm 5i$

$r^2 + 25 = 0$

$y = C_1 \cos 5t + C_2 \sin 5t$

(d) $\boxed{1}$ $2y'' - y' - y = 0$

$r_1 = -1/2$, $r_2 = 1$

$2r^2 - r - 1 = 0$

$(2r+1)(r-1) = 0$

$y = C_1 e^{-t/2} + C_2 e^{t}$

(e) $\boxed{1}$ $9y'' - 6y' + y = 0$

$r = 1/3$

$9r^2 - 6r + 1 = 0$

$(3r-1)^2 = 0$

$y = C_1 e^{t/3} + C_2 t e^{t/3}$

(f) $\boxed{1}$ $y'' - 2y' + 10y = 0$

$r = 1 \pm 3i$

$r^2 - 2r + 10 = 0$

$(r-1)^2 = -9$

$y = C_1 e^{t} \cos 3t + C_2 e^{t} \sin 3t$

2. Consider the Mass-Spring system given by

$$y'' + 2y' + 2y = 0. \tag{3}$$

(a) $\boxed{1}$ Find a general solution to (3).

$r^2 + 2r + 2 = 0$

$(r+1)^2 = -1$

$r = -1 \pm i$

$y = C_1 e^{-t} \cos t + C_2 e^{-t} \sin t$

(b) $\boxed{1}$ If the Mass-Spring system given in (3) has initial conditions of $y(0) = 2$ and $y'(0) = 1$, find the solution to this initial value problem.

$y(0) = C_1 = 2$

$y'(t) = -C_1 e^{-t} \cos t - C_1 e^{-t} \sin t - C_2 e^{-t} \sin t + C_2 e^{-t} \cos t$

$y'(0) = -C_1 + C_2 = 1$ so $C_2 = 3$

$y = 2e^{-t} \cos t + 3e^{-t} \sin t$

3. $\boxed{2}$ Finding solutions to higher order equations is similar to finding solutions to first and second order equations. Using what we developed for first and second order equations as a starting point, find a general solution to the fifth order equation

$$y^{(5)} = 16y'.$$

$r^5 - 16r = 0$

$r(r^2 - 4)(r^2 + 4) = 0$

$r_1 = 0$

$r_2 = 2$

$r_3 = -2$

$r_4 = 2i$

$r_5 = -2i$

$y = C_1 + C_2 e^{2t} + C_3 e^{-2t} + C_4 \cos 2t + C_5 \sin 2t$