Math 274 Random Laplace Problems

1. Express the following functions using step functions and determine their Laplace transforms.

   (a) $f(t) = \begin{cases} 0, & t < 1 \\ 1, & 1 < t < 2 \\ 2, & 2 < t \end{cases}$

   (b) $g(t) = \begin{cases} t, & t < \pi \\ e^{3t}\cos 2t, & \pi < t \end{cases}$

2. Determine the inverse Laplace transform for the following.

   (a) $H(s) = \dfrac{1}{s} + \dfrac{e^{-2s}}{s^2} + \dfrac{e^{-4s}}{s^3}$

   (b) $K(s) = \dfrac{8e^{4-2s}}{(s-2)(s^2+4)}$

3. Applying the Laplace transform to the initial value problem

$$y'' + 4y = f(t), \qquad y(0) = 0, y'(0) = 0$$

   with

$$f(t) = \begin{cases} 10e^t, & t < 1 \\ -8e^{2t}, & 1 < 2 < t \\ 0, & 2 < t \end{cases}$$

   gives

$$Y(s) = \frac{10}{(s-1)(s^2+4)} + e^{-s}\left[\frac{-8e^2}{(s-2)(s^2+4)} - \frac{10e}{(s-1)(s^2+4)}\right] + e^{-2s}\left[\frac{8e^4}{(s-2)(s^2+4)}\right].$$

   Determine $y(t) = \mathscr{L}^{-1}\{Y(s)\}$, the solution to the given initial value problem.

   You should reference the partial fraction decomposition in 2(b).

4. Apply the Laplace transform to the initial value problem

$$y'' + 5y' - 6y = e^{3t}, \qquad y(0) = 2, y'(0) = 0$$

   to express $Y(s) = \mathscr{L}\{y(t)\}$ in the form $Y(s) = \frac{P(s)}{Q(s)}$; express $P(s)$ as a combined simplified polynomial of the form $P(s) = a_n s^n + a_{n-1}s^{n-1} + \ldots + a_0$ and $Q(s)$ as a fully factored polynomial.

   Do not find the inverse Laplace transform.

5. Assume $g(t)$ is piecewise continuous and of exponential order and consider the initial value problem

$$y' + 3y = g(t), \qquad y(0) = 2.$$

   (a) Find the solution. Express your solution in terms of a convolution.

   (b) Let $g(t) = 3t$, find the solution, i.e. evaluate the convolution integral.

6. Find the solution to the symbolic initial value problem

$$y'' + 2\pi y' + 5\pi^2 y = 4\pi\delta(t-1), \qquad x(0) = 0, x'(0) = 2\pi.$$