1. Express the following functions using step functions and determine their Laplace transforms.

(a) \( f(t) = \begin{cases} 
0, & t < 1 \\
1, & 1 < t < 2 \\
2, & 2 < t 
\end{cases} \)

(b) \( g(t) = \begin{cases} 
t, & t < \pi \\
e^{3t} \cos 2t, & \pi < t 
\end{cases} \)

2. Determine the inverse Laplace transform for the following.

(a) \( H(s) = \frac{1}{s} + \frac{e^{-2s}}{s^2} + \frac{e^{-4s}}{s^3} \)

(b) \( K(s) = \frac{8e^{4-2s}}{(s-2)(s^2 + 4)} \)

3. Applying the Laplace transform to the initial value problem

\[ y'' + 4y = f(t), \quad y(0) = 0, y'(0) = 0 \]

with

\( f(t) = \begin{cases} 
10e^t, & t < 1 \\
-8e^{2t}, & 1 < 2 < t \\
0, & 2 < t 
\end{cases} \)

gives

\[ Y(s) = \frac{10}{(s-1)(s^2 + 4)} + e^{-s} \left[ \frac{-8e^{2}}{(s-2)(s^2 + 4)} - \frac{10e}{(s-1)(s^2 + 4)} \right] + e^{-2s} \left[ \frac{8e^{4}}{(s-2)(s^2 + 4)} \right]. \]

Determine \( y(t) = \mathcal{L}^{-1}\{Y(s)\} \), the solution to the given initial value problem.

You should reference the partial fraction decomposition in 2(b).

4. Apply the Laplace transform to the initial value problem

\[ y'' + 5y' - 6y = e^{3t}, \quad y(0) = 2, y'(0) = 0 \]

to express \( Y(s) = \mathcal{L}\{y(t)\} \) in the form \( Y(s) = \frac{P(s)}{Q(s)} \), express \( P(s) \) as a combined simplified polynomial of the form \( P(s) = a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0 \) and \( Q(s) \) as a fully factored polynomial.

**Do not find the inverse Laplace transform.**

5. Assume \( g(t) \) is piecewise continuous and of exponential order and consider the initial value problem

\( y' + 3y = g(t), \quad y(0) = 2. \)

(a) Find the solution. Express your solution in terms of a convolution.

(b) Let \( g(t) = 3t \), find the solution, i.e. evaluate the convolution integral.

6. Find the solution to the symbolic initial value problem

\[ y'' + 2\pi y' + 5\pi^2 y = 4\pi \delta(t - 1), \quad x(0) = 0, x'(0) = 2\pi. \]