If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible then $\mathbf{A}^{-1} = \frac{1}{|\mathbf{A}|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Variation of Parameters.

$$\mathbf{x}_p(t) = \mathbf{X}(t) \int \mathbf{X}^{-1}(t)\mathbf{f}(t)\,dt$$

$$\mathbf{x}(t) = \mathbf{X}(t)\mathbf{c} + \mathbf{X}(t) \int \mathbf{X}^{-1}(t)\mathbf{f}(t)\,dt$$

$$\mathbf{x}(t) = \mathbf{X}(t)\mathbf{X}^{-1}(t_0)\mathbf{x}_0 + \mathbf{X}(t) \int_{t_0}^{t} \mathbf{X}^{-1}(s)\mathbf{f}(s)\,ds$$

If $\mathbf{X}(t)$ is fundamental matrix for $\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t)$ then

$$e^{\mathbf{At}} = \mathbf{X}(t)\mathbf{X}^{-1}(0).$$

For any 2×2 matrix $\mathbf{A}$, the matrix exponential $e^{\mathbf{At}}$ can be computed according to the table below.

Eigenvalues of $\mathbf{A}$	$e^{\mathbf{At}}$
r_1, r_2 real and distinct	$e^{r_1 t}\frac{1}{r_1 - r_2}(\mathbf{A} - r_2\mathbf{I}) - e^{r_2 t}\frac{1}{r_1 - r_2}(\mathbf{A} - r_1\mathbf{I})$
r real repeated twice	$e^{rt}\mathbf{I} + te^{rt}(\mathbf{A} - r\mathbf{I})$
$\alpha \pm i\beta$ complex conjugate pair	$e^{\alpha t}\cos(\beta t)\mathbf{I} + \frac{1}{\beta}e^{\alpha t}\sin(\beta t)(\mathbf{A} - \alpha\mathbf{I})$