Math 274 | Laplace Thing 1 | Name: ________________

27 Feb 2018 | Show Appropriate Work | Not graded - solutions on webpage.

1. Use the definition to determine the Laplace transform of $f(t) = \begin{cases} t, & 0 \le t < 4 \\ e^{-t}, & 4 < t \end{cases}$

$$\mathcal{L}\{f(t)\} = \int_0^4 t e^{-st}\,dt + \int_4^\infty e^{-st}\cdot e^{-t}\,dt$$

$u = t \quad dv = e^{-st}dt$

$du = dt \quad v = -\frac{1}{s}e^{-st}$

$$= -\frac{t}{s}e^{-st}\Big|_0^4 + \frac{1}{s}\int_0^4 e^{-st}\,dt + \int_4^\infty e^{-(s+1)t}\,dt$$

$$= \frac{-4}{s}e^{-4s} + \frac{1}{s}\left(-\frac{1}{s}e^{-st}\right)\Big|_0^4 + \frac{e^{-(s+1)t}}{-(s+1)}\Big|_4^\infty$$

$$= -\frac{4}{s}e^{-4s} + \frac{1}{s^2} - \frac{1}{s^2}e^{-4s} + \frac{e^{-4(s+1)}}{s+1}$$

2. $\boxed{2}$ Use the provided table and linearity to determine the Laplace transform of the following functions.

(a) $f(t) = 4t^2 - 7e^{2t}\cos 3t$

$$F(s) = 4\cdot\frac{2}{s^3} - 7\frac{(s-2)}{(s-2)^2+9}$$

(b) $g(t) = 6 + t^8 e^{-3t}$

$$G(s) = \frac{6}{s} + \frac{8!}{(s+3)^9}$$

(c) $h(t) = 5te^{2t}\sin 3t$

$$H(s) = -5\frac{d}{ds}\left[\frac{3}{(s-2)^2+9}\right]$$

$$= -5\left[\frac{-6(s-2)}{((s-2)^2+9)^2}\right]$$

$$= \frac{30(s-2)}{((s-2)^2+9)^2}$$

(d) $j(t) = \cos^2 bt$

[HINT: $\cos^2 x = (1+\cos 2x)/2$.]

$$j(t) = \frac{1}{2} + \frac{1}{2}\cos 2bt$$

$$J(s) = \frac{1}{2s} + \frac{1}{2}\cdot\frac{s}{s^2+4b^2}$$

3. For each of the following choose all that apply; the function is not piecewise continuous on $[0, \infty)$ (**NPC**), the function is not of exponential order (**NEO**), and/or the function has a Laplace transform (**LT**).

   (a) **NPC** / **NEO** / **LT** : $f(t) = \tan t$
   (b) **NPC** / **NEO** / **LT** : $f(t) = e^{\sin t}$
   (c) **NPC** / **NEO** / **LT** : $f(t) = 14e^{t^2+7}$
   (d) **NPC** / **NEO** / **LT** : $f(t) = \begin{cases} 4e^{3t}, & 0 < t < 2 \\ 12e^{t}, & 2 < t \end{cases}$

4. The inverse Laplace transform is defined as you would expect it to be[1]. For example, since $\mathscr{L}\{\sin 3t\} = \frac{3}{s^2+9}$, we have $\mathscr{L}^{-1}\left\{\frac{3}{s^2+9}\right\} = \sin 3t$. Find the inverse Laplace transform of the following.

   (a) $F(s) = \dfrac{s}{s^2+4}$

   $f(t) = \cos 2t$

   (b) $G(s) = \dfrac{8s}{s^2+4}$

   $g(t) = 8 \cos 2t$

   (c) $H(s) = \dfrac{6}{s^4}$

   $h(t) = t^3$

   (d) $K(s) = \dfrac{1}{s^4}$

   $k(t) = \frac{1}{6} t^3$

   (e) $W(s) = \dfrac{6-7s}{s^2+9} = 2 \cdot \dfrac{3}{s^2+9} - 7 \cdot \dfrac{s}{s^2+9}$ [HINT: separate the fraction.]

   $w(t) = 2 \sin 3t - 7 \cos 3t$

[1] There are some technical complications we will address next week, but for now we will ignore them and assume it works as we expect.