It is important to understand the correct form of a solution suggested by the Method of Undetermined Coefficients. The following information is given in the class.

Method of Undetermined Coefficients

To find a particular solution to

$$ay'' + by' + cy = P_m(t)e^{rt}$$

where $P_m(t)$ is a polynomial of degree m, use the form

$$y_p(t) = t^s\left(A_m t^m + \cdots + A_1 t + A_0\right)e^{rt};$$

if r is not a root of the associated auxiliary equation, take $s = 0$; if r is a simple root, take $s = 1$; and if r is a double root, take $s = 2$.
To find a particular solution to

$$ay'' + by' + cy = P_m(t)e^{\alpha t}\cos\beta t + Q_n(t)e^{\alpha t}\sin\beta t$$

where $P_m(t)$ and $Q_n(t)$ are polynomials of degree m and n, respectively, use the form

$$y_p(t) = t^s\left(A_k t^k + \cdots + A_1 t + A_0\right)e^{\alpha t}\cos\beta t + t^s\left(B_k t^k + \cdots + B_1 t + B_0\right)e^{\alpha t}\sin\beta t;$$

where k is the larger of m and n. If $\alpha + i\beta$ is not a root of the associated auxiliary equation, take $s = 0$; if so take $s = 1$.

Examples.

Equation	Roots of Ch. Eq	Form
$y'' - y' - 6y = t^2$	$r = 3, -2$	$y_p = At^2 + Bt + C$
$y'' - y' - 6y = t^2e^t$	$r = 3, -2$	$y_p = (At^2 + Bt + C)e^t$
$y'' - y' - 6y = t^2e^{3t}$	$r = 3, -2$	$y_p = t(At^2 + Bt + C)e^{3t}$
$y'' - y' - 6y = t^2e^t + t^2e^{3t}$	$r = 3, -2$	$y_p = (At^2 + Bt + C)e^t + t(Dt^2 + Et + F)e^{3t}$
$y'' - 2y' + 1 = 4t^2e^t$	$r = 1, 1$	$y_p = t^2(At^2 + Bt + C)e^t$
$y'' - 2y' + 1 = 4t^2e^t - 3t$	$r = 1, 1$	$y_p = t^2(At^2 + Bt + C)e^t + Dt + E$
$y'' + 9y = 2t + 1$	$r = \pm 3i$	$y_p = At + B$
$y'' + 9y = \cos 2t$	$r = \pm 3i$	$y_p = A\cos 2t + B\sin 2t$
$y'' + 9y = \cos 3t$	$r = \pm 3i$	$y_p = At\cos 3t + Bt\sin 3t$
$y'' + 9y = 3t\cos 3t$	$r = \pm 3i$	$y_p = (At + B)t\cos 3t + (Ct + D)t\sin 3t$
$y'' + 9y = -7e^t\cos 3t$	$r = \pm 3i$	$y_p = Ae^t\cos 3t + Be^t\sin 3t$
$y'' + 2y' + 5 = 14\cos 2t$	$r = -1 \pm 2i$	$y_p = A\cos 2t + B\sin 2t$
$y'' + 2y' + 5 = -8e^{-2t}\cos 2t$	$r = -1 \pm 2i$	$y_p = Ae^{-2t}\cos 2t + Be^{-2t}\sin 2t$
$y'' + 2y' + 5 = -8e^{-t}\cos 2t$	$r = -1 \pm 2i$	$y_p = Ate^{-t}\cos 2t + Bte^{-t}\sin 2t$

Exercise.

Equation	Roots of Ch. Eq	Form
(1)$y'' - y' - 2y = 2t$		
(2)$y'' - y' - 2y = 2t + 7e^t$		
(3)$y'' - y' - 2y = 2t - 7e^{2t}$		
(4)$y'' - y' - 2y = 2t + \cos 2t$		
(5)$y'' + 4y = 7t^2$		
(6)$y'' + 4y = 3t\cos t$		
(7)$y'' + 4y = 23\sin 2t$		
(8)$y'' + 4y = 7te^t\cos 2t$		

Solutions.

(1) $y_p = At + B$, (2) $y_p = At + B + Ce^t$, (3) $y_p = At + B + Cte^{2t}$, (4) $y_p = At + B + C\cos 2t + D\sin 2t$, (5) $y_p = At^2 + Bt + C$, (6) $y_p = (At + B)\cos t + (Ct + D)\sin t$, (7) $y_p = At\cos 2t + Bt\sin 2t$, (8) $(At + B)e^t\cos 2t + (Ct + D)e^t\sin 2t$