

Math 274

Due: 16 Apr 2019

Matrix Thing
Show Appropriate WorkName: _____
Point Values in boxes.

1. [2] Find a fundamental matrix for the system

$$\mathbf{x}'(t) = \begin{bmatrix} -3 & -2 \\ 3 & 4 \end{bmatrix} \mathbf{x}(t).$$

$$|\bar{A} - r\bar{I}| = \begin{vmatrix} -3-r & -2 \\ 3 & 4-r \end{vmatrix} = (-3-r)(4-r) + 6 = r^2 - r - 6 = (r-3)(r+2)$$

$r = 3$

$r = -2$

$$\begin{bmatrix} -6 & -2 \\ 3 & 1 \end{bmatrix} \vec{u}_1 = \vec{0}, \quad \text{choose } \vec{u}_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -2 \\ 3 & 6 \end{bmatrix} \vec{u}_2 = \vec{0}, \quad \text{choose } \vec{u}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\boxed{\vec{X}} = \begin{bmatrix} e^{3t} & 2e^{-2t} \\ -3e^{3t} & -e^{-2t} \end{bmatrix}$$

2. [3] Find the solution of the initial value problem

$$\begin{aligned} x' &= x - 2y, & x(0) &= 3 \\ y' &= 4x - 3y, & y(0) &= 5. \end{aligned}$$

$$\vec{x}' = \begin{bmatrix} 1 & -2 \\ 4 & -3 \end{bmatrix} \vec{x}, \quad \vec{x}(0) = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\begin{aligned} |\bar{A} - r\bar{I}| &= (1-r)(-3-r) + 8 \\ &= r^2 + 2r + 5 \\ &= (r+1)^2 + 4 \end{aligned}$$

$\text{so } r = -1 \pm 2i$

$\text{Choose } r = -1 + 2i$

$$\begin{bmatrix} 2-2i & -2 \\ 4 & -2-2i \end{bmatrix} \vec{u}_1 = \vec{0}$$

$\text{choose } \vec{u}_1 = \begin{bmatrix} 1 \\ 1-i \end{bmatrix}$

A general solution is

$$\vec{x} = C_1 e^{-t} \left(\cos 2t \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \sin 2t \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right) + C_2 e^{-t} \left(\sin 2t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \cos 2t \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right)$$

$$\vec{x}(0) = C_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$\text{so } C_1 = 3, C_2 = -2$

$$\vec{x} = 3e^{-t} \left(\cos 2t \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \sin 2t \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right)$$

$$-2e^{-t} \left(\sin 2t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \cos 2t \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right)$$

3. [5] Find a fundamental solution set for the system

$$\begin{aligned}x' &= x + 2y - z \\y' &= y + z \\z' &= -y + z.\end{aligned}$$

$$\vec{x} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} \vec{x}$$

A fundamental solution set is then

$$|\vec{A} - r\vec{I}| = (1-r) \left[(1-r)^2 + 1 \right] = 0$$

$$\left\{ \begin{bmatrix} e^t \\ 0 \\ 0 \end{bmatrix}, e^t \begin{bmatrix} 2 \cos t + \sin t \\ -\sin t \\ -\cos t \end{bmatrix}, e^t \begin{bmatrix} 2 \sin t - \cos t \\ \cos t \\ -\sin t \end{bmatrix} \right\}$$

$$r_1 = 1, \quad r_2 = 1 \pm i$$

$$r_1 = 1 : \begin{bmatrix} 0 & 2 & -1 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \vec{u}_1 = \vec{0}$$

$$\text{choose } \vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$r_2 = 1+i : \begin{bmatrix} -i & 2 & -1 \\ 0 & -i & 1 \\ 0 & -1 & -i \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \vec{0}$$

$$-i u_2 + u_3 = 0, \quad \text{choose } u_2 = -i \\ u_3 = 1$$

$$-i u_1 + 2 u_2 - u_3 = 0$$

$$-i u_1 + 2(-i) - 1 = 0$$

$$u_1 + 2 - i = 0$$

$$u_1 = i - 2$$

$$\text{so } \vec{u}_2 = \begin{bmatrix} i-2 \\ -i \\ 1 \end{bmatrix} \text{ works, so does } \vec{u}_3 = \begin{bmatrix} 2-i \\ i \\ -1 \end{bmatrix}$$