

Quiz 3

Show Appropriate Work

Name: _____
Point Values in

1. Find a general solution for the following.

(a) 2 $x'' + 3x' + 2x = 0$

$$r^2 + 3r + 2 = 0$$

$$(r+1)(r+2) = 0$$

$$r_1 = -1, r_2 = -2$$

$$x(t) = C_1 e^{-t} + C_2 e^{-2t}$$

(b) 2 $y'' + 2y' + 1y = 0$

$$y(t) = C_1 e^{-t} + C_2 t e^{-t}$$

$$r^2 + 2r + 1 = 0$$

$$(r+1)^2 = 0$$

$$r = -1, \text{ repeated}$$

(c) 2 $z'' + 2z' + 2z = 0$

$$z(t) = C_1 e^{-t} \cos t + C_2 e^{-t} \sin t$$

$$r^2 + 2r + 2 = 0$$

$$(r+1)^2 = -1$$

$$r = -1 \pm i$$

2. Consider the mass-spring system given by

$$y'' + 6y' + 8y = 0.$$

The system has initial data $y(0) = -1$ and $y'(0) = -2$. Solve the initial value problem for this system.

$$r^2 + 6r + 8 = 0$$

$$(r+2)(r+4) = 0$$

$$r_1 = -2, r_2 = -4$$

$$2C_1 + 2C_2 = -2$$

$$-2C_1 - 4C_2 = -2$$

$$\underline{-2C_1 = -4 \quad \text{so} \quad C_2 = 2}$$

$$y = C_1 e^{-2t} + C_2 e^{-4t}$$

$$C_1 = -3$$

$$y(0) = C_1 + C_2 = -1$$

$$y = 2e^{-4t} - 3e^{-2t}$$

$$y' = -2C_1 e^{-2t} - 4C_2 e^{-4t}$$

$$y'(0) = -2C_1 - 4C_2 = -2$$