

## Quiz 4

Show Appropriate Work

Name: \_\_\_\_\_

Scaled to 10 points.

1. 6 Find the form of a particular solution suggested by the Method of Undetermined Coefficients for the following. **Do not** solve for the coefficients.

(a)  $y'' - y' - 6y = e^{3t} \cos 2t + 7te^{-2t}$

$$r^2 - r - 6 = 0$$

$$(r-3)(r+2) = 0$$

$$r_1 = 3 \quad y_1 = e^{3t}$$

$$r_2 = -2 \quad y_2 = e^{-2t}$$

$$y_p = Ae^{3t} \cos 2t + Be^{3t} \sin 2t + (Ct + D)e^{-2t}$$

(b)  $y'' + y = t \cos 3t$

$$y_p = (At + B) \cos 3t + (Ct + D) \sin 3t$$

$$r^2 + 1 = 0$$

$$r = \pm i$$

$$y_1 = \cos t$$

$$y_2 = \sin t$$

2. 4 Find  $A, B, C$  so that  $y_p = At + B + Ce^{3t}$  solves

$$y'' - 2y' + 3y = 6t + 3e^{3t}.$$

$$y_p = At + B + Ce^{3t}$$

$$y_p' = A + 3Ce^{3t}$$

$$y_p'' = 9Ce^{3t}$$

$$y_p'' - 2y_p' + 3y_p = 9Ce^{3t} - 2A - 6Ce^{3t} + 3At + 3B + 3Ce^{3t}$$

$$= 6Ce^{3t} + 3At + (3B - 2A)$$

$$= 3e^{3t} + 6t + 0$$

$$\text{so } C = \frac{1}{2}, \quad A = 2, \quad \text{and } 3B - 2A = 0$$

$$\text{so } B = \frac{4}{3}$$

[CONTINUED ON THE REVERSE.]

3. [4] Find a general solution to the following.

(a)  $t^2y'' - 2y = 0$

$$\begin{aligned} r^2 - r - 2 &= 0 \\ (r-2)(r+1) &= 0 \\ r_1 &= 2 \quad \text{or} \quad r_2 = -1 \end{aligned}$$

$$y = C_1 t^2 + C_2 t^{-1}$$

(b)  $t^2y'' + 5ty' + 4y = 0$

$$\begin{aligned} r^2 + 4r + 4 &= 0 \\ (r+2)^2 &= 0 \\ r &= -2, \text{ repeated} \end{aligned}$$

$$y = C_1 t^{-2} + C_2 t^{-2} \ln t$$

### Reduction of Order

If  $y_1(t)$  is a solution, not identically zero, to  $y'' + p(t)y' + q(t)y = 0$  on  $I$ , then

$$y_2(t) = y_1(t) \int \frac{e^{-\int p(t) dt}}{(y_1(t))^2} dt$$

is a second, linearly independent solution.

4. [6] For  $x > 0$ , find a second linearly independent solution to

$$xy'' - (x+1)y' + y = 0$$

provided that  $y_1 = e^x$  is a solution.

$$p(x) = -\left(\frac{x+1}{x}\right) = -1 - \frac{1}{x}$$

$$\begin{aligned} y_2 &= e^x \int \frac{e^{\int (1 + \frac{1}{x}) dx}}{e^{2x}} dx = e^x \int \frac{e^{x + \ln x}}{e^{2x}} dx = e^x \int \frac{e^x \cdot x}{e^{2x}} dx \\ &= e^x \int x e^{-x} dx = e^x \left[ -x e^{-x} - \int -e^{-x} dx \right] = e^x \left[ -x e^{-x} - e^{-x} \right] \\ u &= x \quad dv = e^{-x} dx \quad = -x - 1 \\ du &= dx \quad v = -e^{-x} \end{aligned}$$