Math 274 | Quiz 5 | Name: _____

23 Mar 2018 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{3}$ Express the following function using step functions and determine its Laplace transforms.

$$f(t) = \begin{cases} 1, & t < 2 \\ t, & 2 < t < 3 \\ e^{2t}, & 3 < t \end{cases}$$

$$= 1 + u(t-2)(t-1) + u(t-3)(e^{2t} - t)$$

$$F(s) = \frac{1}{s} + e^{-2s}\mathcal{L}\{t+1\} + e^{-3s}\mathcal{L}\{e^{2t+6} - t - 3\}$$

$$= \frac{1}{s} + e^{-2s}\left(\frac{1}{s^2} + \frac{1}{s}\right) + e^{-3s}\left[e^{6}\cdot\frac{1}{s-2} - \frac{1}{s^2} - \frac{3}{s}\right]$$

2. $\boxed{7}$ Applying the Laplace transform to the initial value problem

$$y'' + 4y = \begin{cases} 0, & t < 2 \\ 8e^t, & 2 < t \end{cases}, \qquad y(0) = 1, y'(0) = 2$$

gives

$$Y(s) = \frac{s+2}{s^2+4} + \frac{8e^{4-2s}}{(s-2)(s^2+4)}.$$

Determine $y(t) = \mathscr{L}^{-1}\{Y(s)\}$, the solution to the given initial value problem.

$$\frac{8}{(s-2)(s^2+4)} = \frac{A}{s-2} + \frac{Bs+C}{s^2+4}$$

$$8 = A(s^2+4) + (Bs+C)(s-2)$$

$s-2$: $8 = A(8)$ so $A = 1$

s^2: $0 = A + B$ so $B = -1$

s^0: $8 = 4A - 2C$ so $C = -2$

$$y(t) = \cos 2t + \sin 2t + e^4 u(t-2)\left[e^{2(t-2)} - \cos(2t-4) - \sin(2t-4)\right]$$