Math 274 | Quiz 6 | Name: ______________
28 Mar 2018 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. Assume $g(t)$ is piecewise continuous and of exponential order and consider the initial value problem

$$y'' + 2y' + y = g(t), \qquad y(0) = 1, y'(0) = 2.$$

(a) $\boxed{4}$ Find the solution. Express your solution in terms of a convolution.

$$s^2Y - s - 2 + 2sY - 2 + Y = G(s)$$

$$Y(s^2 + 2s + 1) = G + s + 4$$

$$Y = G\left(\frac{1}{(s+1)^2}\right) + \frac{s+4}{(s+1)^2} \longleftarrow \text{Note: } \frac{s+4}{(s+1)^2} = \frac{s+1}{(s+1)^2} + \frac{3}{(s+1)^2}$$

$$y = g(t) * te^{-t} + e^{-t} + 3te^{-t}$$

(b) $\boxed{1}$ Express the convolution in part (a) as an appropriate integral.

$$g(t) * te^{-t} = \int_0^t g(t-v)\, v e^{-v}\, dv$$

CONTINUED ON REVERSE.

2. Consider a mass-spring system sitting in front of a cuckoo clock. After π seconds the time is exactly 1 pm. The cuckoo comes out of the clock and strikes the system exerting an impulse on the mass. The system is governed by the symbolic initial value problem

$$x'' + 4x = 2\delta(t - \pi), \qquad x(0) = 0, x'(0) = -2, \tag{1}$$

where $x(t)$ measures the displacement from the equilibrium.

(a) $\boxed{4}$ Determine $x(t)$, i.e. solve the symbolic initial value problem (1).

$$s^2 X + 2 + 4X = 2e^{-\pi s}$$

$$X(s^2+4) = -2 + 2e^{-\pi s}$$

$$X = \frac{-2}{s^2+4} + \frac{2}{s^2+4} e^{-\pi s}$$

$$x = -\sin 2t + u(t-\pi)\underbrace{\sin(2(t-\pi))}_{\sin(2t-2\pi) = \sin 2t}$$

$$x = -\sin 2t + u(t-\pi)\sin 2t$$

$$= \begin{cases} -\sin 2t & 0 < t < \pi \\ 0 & \text{otherwise} \end{cases}$$

(b) $\boxed{1}$ Carefully sketch a graph of $x(t)$ for $t \in [0, 2\pi]$.