

Math 274
Due: 19 Mar 2018

Inverse Laplace Thing
Show Appropriate Work

Name: _____
Scaled to 10.

1. Determine the inverse Laplace transform of the following.

(a) 1 $F(s) = \frac{4}{s+3}$

(d) 1 $J(s) = \frac{2}{(s-3)^4} = \frac{2}{3!} \cdot \frac{3!}{(s-3)^4}$

$$f(t) = 4e^{-3t}$$

$$j(t) = \frac{1}{3}t^3 e^{3t}$$

(b) 1 $G(s) = \frac{3}{2s+1} = \frac{\frac{3}{2}}{s + \frac{1}{2}}$

(e) 1 $K(s) = \frac{2}{s^2+3} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{s^2+3}$

$$g(t) = \frac{3}{2}e^{-t/2}$$

$$k(t) = \frac{2}{\sqrt{3}} \sin(\sqrt{3}t)$$

(c) 1 $H(s) = \frac{2}{3-s} = \frac{-2}{s-3}$

(f) 1 $M(s) = \frac{2s}{s^2+3}$

$$h(t) = -2e^3 t$$

$$m(t) = 2 \cos(\sqrt{3}t)$$

(g) 1 $N(s) = \frac{2s+2}{s^2+3}$

$$n(t) = \frac{2}{\sqrt{3}} \sin(\sqrt{3}t) + 2 \cos(\sqrt{3}t)$$

(h) 1 $P(s) = \frac{6s}{s^2+4s+6} = \frac{L(s+2)}{(s+2)^2 + 2}$

$$p(t) = 6e^{-2t} \cos(\sqrt{2}t) - \frac{12}{\sqrt{2}} e^{-2t} \sin(\sqrt{2}t)$$

(i) 4 $Q(s) = \frac{4s+2}{s^3+2s^2} = \frac{\frac{3}{2}}{s} + \frac{1}{s^2} - \frac{\frac{3}{2}}{s+2}$

$$q(t) = \frac{3}{2} + t - \frac{3}{2} e^{-2t}$$

2. Consider the initial value problem

$$y'' + 4y = 8t - 4, \quad y(0) = 1, y'(0) = 0. \quad (1)$$

(a) [6] Applying the Laplace transform to the initial value problem (1) gives the following

$$[s^2Y(s) - s] + 4Y(s) = \frac{8}{s^2} - \frac{4}{s}. \quad (2)$$

Solve equation (2) above for $Y(s)$ and then determine $y(t) = \mathcal{L}^{-1}\{Y(s)\}$ which is the solution to the initial value problem.

$$Y \left[\frac{s^2 + 4}{s^2} \right] = \frac{8 - 4s}{s^2} + \frac{4}{s} = \frac{8 - 4s + s^3}{s^2 (s^2 + 4)}$$

so

$$Y = \frac{8 - 4s + s^3}{s^2 (s^2 + 4)} = \frac{-1}{s} + \frac{2}{s^2} + \frac{2s}{s^2 + 4} - \frac{2}{s^2 + 4}$$

$$y = -1 + 2t + 2\cos 2t - \sin 2t$$

(b) [2] Use methods from Chapter 4 to solve the the initial value problem (1).

$$r^2 + 4 = 0$$

$$y = C_1 \cos 2t + C_2 \sin 2t + 2t - 1$$

so

$$y(0) = 1 \Rightarrow C_1 = 2$$

$$y = C_1 \cos 2t + C_2 \sin 2t$$

$$y' = -4\sin 2t + 2C_2 \cos 2t + 2$$

$$y'(0) = 0 \Rightarrow C_2 = -1$$

$$y_p = At + B$$

$$y_p' = A$$

$$y_p'' = 0$$

$$\text{Substituting gives } 4At + 4B = 8t - 4$$

$$\text{so } A = 2, B = -1$$

$$y = 2\cos 2t - \sin 2t + 2t - 1$$