Math 274 | IVP Laplace Thing | Name: _____

Due: 20 Mar 2018 | Show Appropriate Work | Scaled to [10].

1. [6] Applying the Laplace transform to the initial value problem

$$y'' + 2y' + y = 27e^{2t}, \qquad y(0) = 4, y'(0) = 3$$

gives the following

$$Y(s) = \frac{4s^2 + 3s + 5}{(s^2 + 2s + 1)(s - 2)}. \tag{1}$$

Determine $y(t) = \mathscr{L}^{-1}\{Y(s)\}$, the solution to the given initial value problem.

$$\frac{4s^2 + 3s + 5}{(s^2 + 2s + 1)(s-2)} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{s-2}$$

$$4s^2 + 3s + 5 = A(s+1)(s-2) + B(s-2) + C(s+1)^2$$

$s = 2$: $16 + 8 + 5 = 9C$ so $C = 3$

$s = -1$: $4 - 3 + 5 = B(-3)$ so $B = -2$

s^2: $4 = A + C$ so $A = 1$

$$Y(s) = \frac{1}{s+1} - \frac{2}{(s+1)^2} + \frac{3}{s-2}$$

$$y(t) = e^{-t} - 2te^{-t} + 3e^{2t}$$

2. [6] Apply the Laplace transform to the initial value problem

$$y'' + 3y' + y = t\cos 2t, \qquad y(0) = 1, y'(0) = -3$$

to express $Y(s) = \mathscr{L}\{y(t)\}$ in the form $Y(s) = \frac{P(s)}{Q(s)}$; for example, (1) above is of this form. **Do not find the inverse Laplace transform.**

$$s^2Y - s + 3 + 3(sY - 1) + Y = -\frac{d}{ds}\left(\frac{s}{s^2+4}\right) = -\left(\frac{s^2 + 4 - 2s^2}{(s^2+4)^2}\right)$$

$$Y(s^2 + 3s + 1) = \frac{s^2 - 4}{(s^2+4)^2} + \frac{s(s^4 + 8s^2 + 16)}{(s^2+4)^2}$$

$$Y(s) = \frac{s^5 + 8s^3 + s^2 + 16s - 4}{(s^2+4)^2(s^2+3s+1)}$$

3. [8] Solve the initial value problem using the method of Laplace transforms

$$(t+1)y'' - 2y' = -1, \qquad y(0) = y'(0) = 0.$$

$$-\frac{d}{ds}\left[s^2 Y\right] + s^2 Y - 2sY = -\frac{1}{s}$$

$$-2sY - s^2 Y' + s^2 Y - 2sY = -\frac{1}{s}$$

$$-s^2 Y' + (s^2 - 4s)Y = -\frac{1}{s}$$

$$Y' + \left(\frac{4}{s} - 1\right)Y = \frac{1}{s^3} \quad \longleftarrow \text{This is first order linear in std. form}$$

$$u(s) = e^{4\ln s - s} = s^4 e^{-s}, \text{ so}$$

$$Y = \frac{e^s}{s^4}\int s e^{-s}\, ds \quad \searrow \text{Integration by Parts}$$

$$= \frac{e^s}{s^4}\left[-se^{-s} - e^{-s} + C\right]$$

$$= -\frac{1}{s^3} - \frac{1}{s^4} + \frac{Ce^s}{s^4}$$

Since $Y \xrightarrow[s\to\infty]{} 0$, $C = 0$

$$Y = -\frac{1}{s^3} - \frac{1}{s^4}$$

so

$$y(t) = -\frac{1}{2}t^2 - \frac{1}{6}t^3$$