Math 274 | Quiz 1 | Name: _____

19 Jan 2018 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. Consider the differential equation

$$\frac{dy}{dt} = y^2 - 1. \tag{1}$$

(a) $\boxed{3}$ Verify $\phi(t) = \dfrac{1+ce^{2t}}{1-ce^{2t}}$ is a one parameter family of solutions to (1) for any constant c.

$$\varphi' = \frac{2ce^{2t}(1-ce^{2t}) - (1+ce^{2t})(-2ce^{2t})}{(1-ce^{2t})^2}$$

$$= \frac{2ce^{2t} - 2c^2e^{4t} + 2ce^{2t} + 2c^2e^{4t}}{(1-ce^{2t})^2}$$

$$= \frac{4ce^{2t}}{(1-ce^{2t})^2}$$

$$\varphi^2 - 1 = \frac{1+2ce^{2t}+c^2e^{4t}}{(1-ce^{2t})^2} - 1$$

$$= \frac{1+2ce^{2t}+c^2e^{4t} - (1-ce^{2t})^2}{(1-ce^{2t})^2}$$

$$= \frac{4ce^{2t}}{(1-ce^{2t})^2}$$

Since $\varphi' = \varphi^2 - 1$, φ is a solution to (1).

(b) $\boxed{1}$ Verify that for $c = 2$ the solution above, i.e. $\phi(t) = \dfrac{1+2e^{2t}}{1-2e^{2t}}$, satisfies the initial data $y(0) = -3$.

$$\varphi(0) = \frac{1+2}{1-2} = -3 \quad \checkmark$$

(c) $\boxed{1}$ Is the solution in (b) above the only solution to (1) that satisfies $y(0) = -3$? Justify your solution.

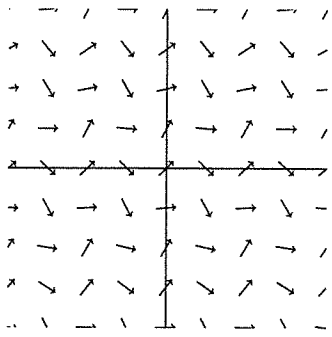
Yes, since y^2-1 & $\frac{\partial}{\partial y}(y^2-1) = 2y$ are both continuous everywhere, there is a <u>unique</u> solution satisfying $y(0) = -3$.

(d) $\boxed{1}$ What are the equilibrium solutions to (1), i.e. solutions of the form $y \equiv C$ for some constant C?

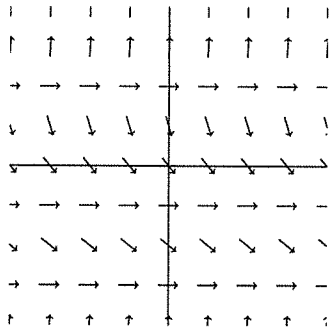
$y \equiv 1$ & $y \equiv -1$

[CONTINUED ON THE REVERSE.]

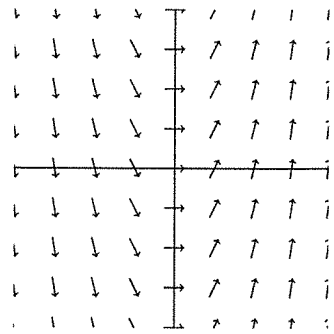
2. $\boxed{2}$ Label each direction field below as either **autonomous** or **not autonomous**. Circle the appropriate choice below each.



Autonomous :: Not



Autonomous :: Not



Autonomous :: Not

3. $\boxed{2}$ The direction field for $\dfrac{dy}{dx} = x - y$ is plotted below. Sketch the trajectories, i.e. the solution curves, satisfying the following intial conditions.

(a) $y(0) = -1$

(b) $y(0) = 3$

(c) $y(2) = 0$