1. Consider the differential equation \[
\frac{dy}{dt} = y^2 - 1. \tag{1}
\]

(a) \[3\] Verify \(\phi(t) = \frac{1 + ce^{2t}}{1 - ce^{2t}}\) is a one parameter family of solutions to (1) for any constant \(c\).

\[
\begin{align*}
\phi' &= \frac{2ce^{2t}(1-ce^{2t}) - (1+ce^{2t})(-2ce^{2t})}{(1-ce^{2t})^2} \\
&= \frac{2ce^{2t} - 2e^{4t} + 2ce^{2t} + 2e^{4t}}{(1-ce^{2t})^2} \\
&= \frac{4ce^{2t}}{(1-ce^{2t})^2}
\end{align*}
\]

Since \(\phi' = \phi^2 - 1\), \(\phi\) is a solution to (1).

(b) \[1\] Verify that for \(c = 2\) the solution above, i.e. \(\phi(t) = \frac{1 + 2e^{2t}}{1 - 2e^{2t}}\), satisfies the initial data \(y(0) = -3\).

\[
\phi(0) = \frac{1 + 2}{1 - 2} = -3
\]

(c) \[1\] Is the solution in (b) above the only solution to (1) that satisfies \(y(0) = -3\)? Justify your solution.

Yes, since \(y^2 - 1 \leq \frac{4}{3} (y^2 - 1) = 2y\) are both continuous everywhere, there is a unique solution satisfying \(y(0) = -3\).

(d) \[1\] What are the equilibrium solutions to (1), i.e. solutions of the form \(y \equiv C\) for some constant \(C\)?

\[
y \equiv 1 \quad \text{or} \quad y \equiv -1
\]

[CONTINUED ON THE REVERSE.]
2. Label each direction field below as either autonomous or not autonomous. Circle the appropriate choice below each.

3. The direction field for \( \frac{dy}{dx} = x - y \) is plotted below. Sketch the trajectories, i.e. the solution curves, satisfying the following initial conditions.

(a) \( y(0) = -1 \)
(b) \( y(0) = 3 \)
(c) \( y(2) = 0 \)