Math 274 | Quiz 2 | Name: _____

26 Jan 2018 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{2}$ The equation

$$(x+y+7)dx - (\cos(y) - x)dy = 0$$

is exact. Find an implicit general solution.

$$(x+y+7)\,dx + (x - \cos y)\,dy = 0$$

$$\frac{x^2}{2} + xy + 7x - \sin y = C$$

2. $\boxed{4}$ Find an implicit general solution to

$$\frac{dy}{dt} = \frac{te^t(y^2+1)}{y}.$$

$$\int \frac{y\,dy}{y^2+1} = \int te^t\,dt$$

$u = t \qquad dv = e^t\,dt$

$du = dt \qquad v = e^t$

$$\frac{1}{2}\ln(y^2+1) = te^t - e^t + C$$

[Continued on the reverse.]

3. $\boxed{4}$ On Wednesday I intoduced Bernoulli equations. The equation

$$\frac{dy}{dx} - \frac{y}{2x} = \left(\frac{x}{y}\right)^3 \tag{1}$$

is Bernoulli. After applying the substitution $v = y^4$ to (1) we have the new equation

$$\frac{1}{4}\frac{dv}{dx} - \frac{v}{2x} = x^3. \tag{2}$$

Find an explicit general solution to (2).

$$\frac{dv}{dx} - \frac{2}{x} v = 4x^3$$

$$\mu(x) = e^{\int -\frac{2}{x} dx} = e^{-2 \ln|x|} = x^{-2}$$

choose

so

$$v = x^2 \int 4x \, dx$$

$$= x^2 \left(2x^2 + C\right)$$

Bonus

so $y = \pm \left[ 2x^4 + Cx^2 \right]^{1/4}$

solves (1).