1. Assume $g(t)$ is piecewise continuous and of exponential order and consider the initial value problem

$$y'' + 2y' + y = g(t), \quad y(0) = 1, y'(0) = 2.$$ 

(a) Find the solution. Express your solution in terms of a convolution.

(b) Express the convolution in part (a) as an appropriate integral.
2. Consider a mass-spring system sitting in front of a cuckoo clock. After \( \pi \) seconds the time is exactly 1 pm. The cuckoo comes out of the clock and strikes the system exerting an impulse on the mass. The system is governed by the symbolic initial value problem

\[
x'' + 4x = 2\delta(t - \pi), \quad x(0) = 0, x'(0) = -2,
\]

where \( x(t) \) measures the displacement from the equilibrium.

(a) Determine \( x(t) \), i.e. solve the symbolic initial value problem (1).

(b) Carefully sketch a graph of \( x(t) \) for \( t \in [0, 2\pi] \).