1. For $t > 0$, consider the following

$$A(t) = \begin{bmatrix} 0 & 1 \\ -1/t & (t + 1)/t \end{bmatrix}, \quad x_1(t) = \begin{bmatrix} e^t \\ e^t \end{bmatrix}, \quad \text{and} \quad x_2(t) = \begin{bmatrix} t + 1 \\ 1 \end{bmatrix}. $$

(a) Show \{x_1, x_2\} is a fundamental solution set\(^1\) for $x' = Ax$.

(b) Find the solution to the initial value problem $x' = Ax, x(1) = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$.

\(^1\)Show: (i) $x_1$ and $x_2$ are solutions, and (ii) they are linearly independent. (Use the Wronskian.)
2. Two tanks are initially filled with 1 kL of pure water. A solution with 10 kg/kL of salt is flowing into tank 1 at 5 kL/hr. A solution with 20 kg/kL of salt is flowing into tank 2 at 5 kL/hr. Both tanks are well mixed. The resulting solution is flowing from tank 1 into tank 2 at 3 kL/hr, and from tank 2 into tank 1 at 2 kL/hr. Tank 1 is being drained at 4 kL/hr and tank 2 is being drained at 6 kL/hr. Let \( x(t) \) be the amount of salt in tank 1 in kg, and \( y(t) \) be the amount of salt in tank 2 in kg.

(a) Set up an initial value problem that models the amount of salt in each tank.

\[
\begin{bmatrix}
  x'(t) \\
  y'(t)
\end{bmatrix} = \]

Identify the \( x \)-nullcline(s), the \( y \)-nullcline(s), and any equilibrium\(^2\).

Carefully sketch the phase plane for this system for \([0, 50] \times [0, 50]\). Include the nullclines (with direction arrows) and equilibrium you found above. Also include the solution curves that satisfy the initial data \([0, 0]^T\) and \([40, 20]^T\).

In a sentence or two, explain what the equilibrium solution means in this system.

\(^2\)Your equilibrium solution should have integer values for each component.