1. For $t > 0$, consider the following

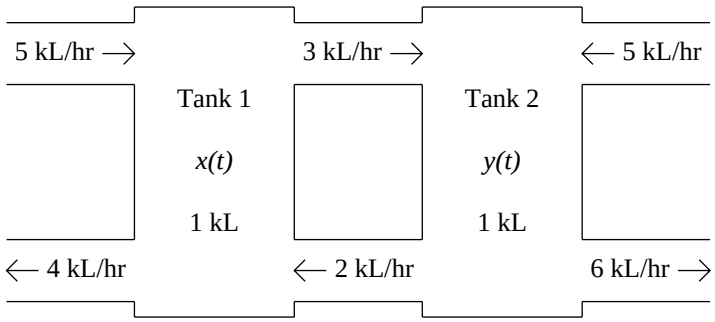
$$\mathbf{A}(t) = \begin{bmatrix} 0 & 1 \\ -1/t & (t+1)/t \end{bmatrix}, \mathbf{x}_1(t) = \begin{bmatrix} e^t \\ e^t \end{bmatrix}, \text{ and } \mathbf{x}_2(t) = \begin{bmatrix} t+1 \\ 1 \end{bmatrix}.$$

(a) Show $\{\mathbf{x}_1, \mathbf{x}_2\}$ is a fundamental solution set[1] for $\mathbf{x}' = \mathbf{A}\mathbf{x}$.

(b) Find the solution to the initial value problem $\mathbf{x}' = \mathbf{A}\mathbf{x}, \mathbf{x}(1) = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$.

[1]Show: (i) $\mathbf{x}_1$ and $\mathbf{x}_2$ are solutions, and (ii) they are linearly independent. (Use the Wronskian.)

2. Two tanks are initially filled with 1 kL of pure water. A solution with 10 kg/kL of salt is flowing into tank 1 at 5 kL/hr. A solution with 20 kg/kL of salt is flowing into tank 2 at 5 kL/hr. Both tanks are well mixed. The resulting solution is flowing from tank 1 into tank 2 at 3 kL/hr, and from tank 2 into tank 1 at 2 kL/hr. Tank 1 is being drained at 4 kL/hr and tank 2 is being drained at 6 kL/hr. Let $x(t)$ be the amount of salt in tank 1 in kg, and $y(t)$ be the amount of salt in tank 2 in kg.

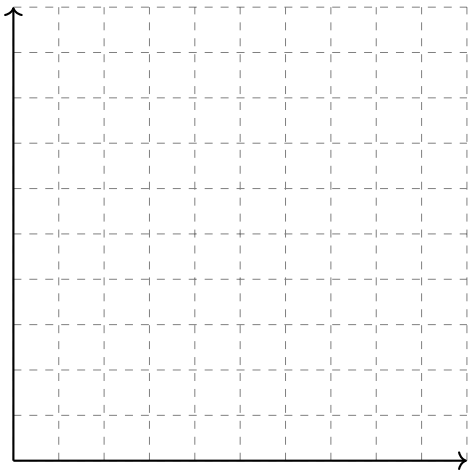


(a) Set up an initial value problem that models the amount of salt in each tank.

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} =$$

Identify the x-nullcline(s), the y-nullcline(s), and any equilibrium[2].

Carefully sketch the phase plane for this system for $[0, 50] \times [0, 50]$. Include the nullclines (with direction arrows) and equilibrium you found above. Also include the solution curves that satisfy the initial data $[0, 0]^T$ and $[40, 20]^T$.



In a sentence or two, explain what the equilibrium solution means in this system.

[2] Your equilibrium solution should have integer values for each component.