Math 274 MUCs Examples and Forms

Section: 4.4

It is important to practice the correct form for the Method of Undetermined Coefficients. Below you will find a number of examples and some exercises for you to try. Do not try to solve for the constants, just specify the form a solution should have.

1. When there is no overlap with solutions to the homogeneous equation we use a form based on the inhomogeneity.

   Examples:

$\lambda_1 = 3,\ \lambda_2 = -2$

(a) $y'' - y' - 6y = 5$	$y_p = A$
(b) $y'' - y' - 6y = 5t$	$y_p = At + B$
(c) $y'' - y' - 6y = 5t^2$	$y_p = At^2 + Bt + C$
(d) $y'' - y' - 6y = e^{5t}$	$y_p = Ae^{5t}$
(e) $y'' - y' - 6y = (t+3)e^{5t}$	$y_p = (At + B)e^{5t}$
(f) $y'' - y' - 6y = \cos 5t$	$y_p = A\cos 5t + B\sin 5t$
(g) $y'' - y' - 6y = t\cos 5t$	$y_p = (At + B)\cos 5t + (Ct + D)\sin 5t$
(h) $y'' - y' - 6y = e^{3t}\cos 5t$	$y_p = Ae^{3t}\cos 5t + Be^{3t}\sin 5t$
$\lambda = \pm 5i \longrightarrow$ (i) $y'' + 25y = e^{3t}\cos 5t$	$y_p = Ae^{3t}\cos 5t + Be^{3t}\sin 5t$

   Exercises:

$\lambda_1 = -4,\ \lambda_2 = 2$

(a) $y'' + 2y' - 8y = 7e^{3t}$ $\quad y_p = Ae^{3t}$

(b) $y'' + 2y' - 8y = 32t$ $\quad y_p = At + B$

(c) $y'' + 2y' - 8y = 7e^{3t} + 32t$ $\quad y_p = Ae^{3t} + (Bt + C)$ — combine

(d) $y'' + 2y' - 8y = 40\cos 2t$ $\quad y_p = A\cos 2t + B\sin 2t$

$\lambda = \pm 5i$

(e) $y'' + 25y = e^{5t}$ $\quad y_p = Ae^{5t}$

(f) $y'' + 25y = e^t \sin 5t$ $\quad y_p = Ae^t \cos 5t + Be^t \sin 5t$

(g) $y'' + 25y = t^2 - 7$ $\quad y_p = At^2 + Bt + C$

(h) $y'' + 25y = \sin t + \cos 2t$ $\quad y_p = A\cos t + B\sin t + C\cos 2t + D\sin 2t$

$\lambda = -1$, repeated

(i) $y'' + 2y' + y = (3t + 4)e^{2t}$ $\quad y_p = (At + B)e^{2t}$

(j) $y'' + 2y' + y = 7t + \cos t$ $\quad y_p = At + B + C\cos t + D\sin t$

(k) $y'' + 2y' + y = 7t\sin t$ $\quad y_p = (At + B)\cos t + (Ct + D)\sin t$

(l) $y'' + 2y' + y = 2te^{-t}\sin t$ $\quad y_p = (At + B)e^{-t}\cos t + (Ct + D)e^{-t}\sin t$

2. When there is overlap with solutions to the homogeneous equation we supplement the form with an extra t, or t^2 in the case of repeated roots.

Examples:

(a) $y'' - y = e^t$	$y_p = Ate^t$
(b) $y'' - y = e^t + te^{3t}$	$y_p = Ate^t + (Bt + C)e^{3t}$
(c) $y'' - y = t^2e^t + 4$	$y_p = t(At^2 + Bt + C)e^t + D$
(d) $y'' + 9y = \cos 3t$	$y_p = At\cos 3t + Bt\sin 3t$
(e) $y'' + 9y = t\sin 3t$	$y_p = t(At + B)\cos 3t + t(Ct + D)\sin 3t$
(f) $y'' - 4y' + 4y = e^{2t}$	$y_p = At^2e^{2t}$
(g) $y'' - 4y' + 4y = (7t^2 + 3)e^{2t} + 3t - 1$	$y_p = t^2(At^2 + Bt + C)e^{2t} + (Dt + E)$

Exercises:

$\lambda_1 = -4$, $\lambda_2 = 2$

(a) $y'' + 2y' - 8y = 12e^{2t}$ $\quad y_p = Ate^{2t}$

(b) $y'' + 2y' - 8y = te^{2t} + 4$ $\quad y_p = t(At+B)e^{2t} + C$

don't add a t to a part that doesn't overlap.

$\lambda = \pm 3i$

(c) $y'' + 9y = \cos 3t$ $\quad y_p = At\cos 3t + Bt\sin 3t$

(d) $y'' + 9y = \sin 3t + \cos 2t$ $\quad y_p = At\cos 3t + Bt\sin 3t + C\cos 2t + D\sin 2t$

(e) $y'' + 9y = t\sin 3t + 3\cos 3t$ $\quad y_p = t(At+B)\cos 3t + t(Ct+D)\sin 3t$

$\lambda = -1$, repeated

(f) $y'' + 2y' + y = e^{-t} + t^2$ $\quad y_p = At^2e^{-t} + (Bt^2 + Ct + D)$

(g) $y'' + 2y' + y = (t+3)e^{-t}$ $\quad y_p = t^2(At+B)e^{-t}$

3. It is often convenient to consider complex exponentials when using the Method of Undetermined Coefficients.

Examples:

(a) $y'' - y' - 6y = 5\sin 2t = \mathrm{Im}(5e^{2ti})$	$z_p = Ae^{2ti}$
(b) $y'' - y' - 6y = 5t\sin 2t = \mathrm{Im}(5te^{2ti})$	$z_p = (At + B)e^{2ti}$
(c) $y'' - y' - 6y = e^t\sin 2t = \mathrm{Im}(e^{(1+2i)t})$	$z_p = Ae^{(1+2i)t}$
(d) $y'' + 9y = \cos 3t = \mathrm{Re}(e^{3it})$	$z_p = Ate^{3it}$

Exercises:

$\lambda = \pm 3i$ (a) $y'' + 9y = \sin 3t = \mathrm{Im}(e^{3ti})$ $\quad z_p = Ate^{3ti}$

$\lambda_1 = -4$, $\lambda_2 = 2$

(b) $y'' + 2y' - 8y = 40\sin 2t = \mathrm{Im}(40e^{2ti})$ $\quad z_p = Ae^{2ti}$

(c) $y'' + 2y' - 8y = 3t\cos 2t = \mathrm{Re}(3te^{2ti})$ $\quad z_p = (At+B)e^{2ti}$

(d) $y'' + 2y' - 8y = e^{2t}\sin 4t = \mathrm{Im}(e^{(2+4i)t})$ $\quad z_p = Ae^{(2+4i)t}$