Math 274 MUCs Examples and Forms

Section: 4.4

It is important to practice the correct form for the Method of Undetermined Coefficients. Below you will find a number of examples and some exercises for you to try. Do not try to solve for the constants, just specify the form a solution should have.

1. When there is no overlap with solutions to the homogeneous equation we use a form based on the inhomogeneity.

   Examples:

   (a) $y'' - y' - 6y = 5$ $\qquad y_p = A$

   (b) $y'' - y' - 6y = 5t$ $\qquad y_p = At + B$

   (c) $y'' - y' - 6y = 5t^2$ $\qquad y_p = At^2 + Bt + C$

   (d) $y'' - y' - 6y = e^{5t}$ $\qquad y_p = Ae^{5t}$

   (e) $y'' - y' - 6y = (t+3)e^{5t}$ $\qquad y_p = (At + B)e^{5t}$

   (f) $y'' - y' - 6y = \cos 5t$ $\qquad y_p = A\cos 5t + B\sin 5t$

   (g) $y'' - y' - 6y = t\cos 5t$ $\qquad y_p = (At + B)\cos 5t + (Ct + D)\sin 5t$

   (h) $y'' - y' - 6y = e^{3t}\cos 5t$ $\qquad y_p = Ae^{3t}\cos 5t + Be^{3t}\sin 5t$

   (i) $y'' + 25y = e^{3t}\cos 5t$ $\qquad y_p = Ae^{3t}\cos 5t + Be^{3t}\sin 5t$

   Exercises:

   (a) $y'' + 2y' - 8y = 7e^{3t}$

   (b) $y'' + 2y' - 8y = 32t$

   (c) $y'' + 2y' - 8y = 7e^{3t} + 32t$

   (d) $y'' + 2y' - 8y = 40\cos 2t$

   (e) $y'' + 25y = e^{5t}$

   (f) $y'' + 25y = e^{t}\sin 5t$

   (g) $y'' + 25y = t^2 - 7$

   (h) $y'' + 25y = \sin t + \cos 2t$

   (i) $y'' + 2y' + y = (3t + 4)e^{2t}$

   (j) $y'' + 2y' + y = 7t + \cos t$

   (k) $y'' + 2y' + y = 7t\sin t$

   (l) $y'' + 2y' + y = 2te^{-t}\sin t$

2. When there is overlap with solutions to the homogeneous equation we supplement the form with an extra t, or t^2 in the case of repeated roots.

   Examples:

   (a) $y'' - y = e^t$ $\qquad y_p = Ate^t$

   (b) $y'' - y = e^t + te^{3t}$ $\qquad y_p = Ate^t + (Bt + C)e^{3t}$

   (c) $y'' - y = t^2e^t + 4$ $\qquad y_p = t(At^2 + Bt + C)e^t + D$

   (d) $y'' + 9y = \cos 3t$ $\qquad y_p = At\cos 3t + Bt\sin 3t$

   (e) $y'' + 9y = t\sin 3t$ $\qquad y_p = t(At + B)\cos 3t + t(Ct + D)\sin 3t$

   (f) $y'' - 4y' + 4y = e^{2t}$ $\qquad y_p = At^2e^{2t}$

   (g) $y'' - 4y' + 4y = (7t^2 + 3)e^{2t} + 3t - 1$ $\qquad y_p = t^2(At^2 + Bt + C)e^{2t} + (Dt + E)$

   Exercises:

   (a) $y'' + 2y' - 8y = 12e^{2t}$

   (b) $y'' + 2y' - 8y = te^{2t} + 4$

   (c) $y'' + 9y = \cos 3t$

   (d) $y'' + 9y = \sin 3t + \cos 2t$

   (e) $y'' + 9y = t\sin 3t + 3\cos 3t$

   (f) $y'' + 2y' + y = e^{-t} + t^2$

   (g) $y'' + 2y' + y = (t + 3)e^{-t}$

3. It is often convenient to consider complex exponentials when using the Method of Undetermined Coefficients.

   Examples:

   (a) $y'' - y' - 6y = 5\sin 2t = \mathrm{Im}(5e^{2ti})$ $\qquad z_p = Ae^{2ti}$

   (b) $y'' - y' - 6y = 5t\sin 2t = \mathrm{Im}(5te^{2ti})$ $\qquad z_p = (At + B)e^{2ti}$

   (c) $y'' - y' - 6y = e^t\sin 2t = \mathrm{Im}(e^{(1+2i)t})$ $\qquad z_p = Ae^{(1+2i)t}$

   (d) $y'' + 9y = \cos 3t = \mathrm{Re}(e^{3it})$ $\qquad z_p = Ate^{3it}$

   Exercises:

   (a) $y'' + 9y = \sin 3t$

   (b) $y'' + 2y' - 8y = 40\sin 2t$

   (c) $y'' + 2y' - 8y = 3t\cos 2t$

   (d) $y'' + 2y' - 8y = e^{2t}\sin 4t$