It is important to practice the correct form for the Method of Undetermined Coefficients. Below you will find a number of examples and some exercises for you to try. Do not try to solve for the constants, just specify the form a solution should have.

1. When there is no overlap with solutions to the homogeneous equation we use a form based on the inhomogeneity.

Examples:

(a) \( y'' - y' - 6y = 5 \)  
(b) \( y'' - y' - 6y = 5t \)  
(c) \( y'' - y' - 6y = 5t^2 \)  
(d) \( y'' - y' - 6y = e^{5t} \)  
(e) \( y'' - y' - 6y = (t + 3)e^{5t} \)  
(f) \( y'' - y' - 6y = \cos 5t \)  
(g) \( y'' - y' - 6y = \sin 5t \)  
(h) \( y'' - y' + 2y = (3t + 4)e^{2t} \)  
(i) \( y'' - y' + y = 2te^{-t} \sin t \)

Exercises:

(a) \( y'' + 2y' - 8y = 7e^{3t} \)  
(b) \( y'' + 2y' - 8y = 32t \)  
(c) \( y'' + 2y' - 8y = 7e^{3t} + 32t \)  
(d) \( y'' + 2y' - 8y = 40 \cos 2t \)  
(e) \( y'' + 25y = e^{5t} \)  
(f) \( y'' + 25y = e^t \sin 5t \)  
(g) \( y'' + 25y = t^2 - 7 \)  
(h) \( y'' + 25y = \sin t + \cos 2t \)  
(i) \( y'' + 2y' + y = (3t + 4)e^{2t} \)  
(j) \( y'' + 2y' + y = 7t + \cos t \)  
(k) \( y'' + 2y' + y = 7t \sin t \)  
(l) \( y'' + 2y' + y = 2te^{-t} \sin t \)
2. When there is overlap with solutions to the homogeneous equation we supplement the form with an extra $t$, or $t^2$ in the case of repeated roots.

Examples:

(a) $y'' - y = e^t$

(b) $y'' - y = e^t + te^{3t}$

(c) $y'' - y = t^2 e^t + 4$

(d) $y'' + 9y = \cos 3t$

(e) $y'' + 9y = t \sin 3t$

(f) $y'' - 4y' + 4y = e^{2t}$

(g) $y'' - 4y' + 4y = (7t^2 + 3)e^{2t} + 3t - 1$

Exercises:

(a) $y'' + 2y' - 8y = 12e^{2t}$

(b) $y'' + 2y' - 8y = te^{2t} + 4$

(c) $y'' + 9y = \cos 3t$

(d) $y'' + 9y = \sin 3t + \cos 2t$

(e) $y'' + 9y = t \sin 3t + 3 \cos 3t$

(f) $y'' + 2y' + y = e^{-t} + t^2$

(g) $y'' + 2y' + y = (t + 3)e^{-t}$

3. It is often convenient to consider complex exponentials when using the Method of Undetermined Coefficients.

Examples:

(a) $y'' - y' - 6y = 5 \sin 2t = \text{Im}(5e^{2it})$

(b) $y'' - y' - 6y = 5t \sin 2t = \text{Im}(5te^{2it})$

(c) $y'' - y' - 6y = e^t \sin 2t = \text{Im}(e^{(1+2i)t})$

(d) $y'' + 9y = \cos 3t = \text{Re}(e^{3it})$

Exercises:

(a) $y'' + 9y = \sin 3t$

(b) $y'' + 2y' - 8y = 40 \sin 2t$

(c) $y'' + 2y' - 8y = 3t \cos 2t$

(d) $y'' + 2y' - 8y = e^{2t} \sin 4t$