Reduction of Order If $y_1(t)$ is a solution, not identically zero, to $y'' + p(t)y' + q(t)y = 0$ on $I$, then

$$y_2(t) = y_1(t) \int \frac{e^{-\int p(t)\,dt}}{(y_1(t))^2} \, dt$$

is a second, linearly independent solution.

1. For $x > 0$, find a general solution to

$$xy'' - y' + (1 - x)y = 0$$

provided that $y_1 = e^x$ is a solution.
Variation of Parameters

If \( y_1 \) and \( y_2 \) are linearly independent solutions to \( y'' + p(t)y' + q(t)y = 0 \), then a particular solution to \( y'' + p(t)y' + q(t)y = g(t) \) is given by
\[
y_p(t) = y_1(t) \int -\frac{g(t)y_2(t)}{W[y_1, y_2](t)} \, dt + y_2(t) \int \frac{g(t)y_1(t)}{W[y_1, y_2](t)} \, dt.
\]

2. Given that a general solution to the homogeneous equation
\[
t^2y'' - 4ty' + 4y = 0.
\]
is \( y = C_1t^4 + C_2t \). Find a particular solution to
\[
t^2y'' - 4ty' + 4y = f(t).
\]
Express your solution as a reasonably simplified combination of integrals, i.e. evaluate the Wronskian and combine like terms. Do not try to integrate an unknown function.