Math 274 | Quiz 1 | Name: ________________
27 June 2017 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{4}$ Determine whether the given relation is an implicit solution to the given differential equation. Assume that the relation does define y as a function of x and use implicit differentiation.

$$e^{xy} + y = x - 1, \qquad \frac{dy}{dx} = \frac{e^{-xy} - y}{e^{-xy} + x}$$

$$\frac{d}{dx}\left[e^{xy} + y\right] = \frac{d}{dx}\left[x - 1\right]$$

$$e^{xy}\left[y + xy'\right] + y' = 1$$

$$e^{xy}xy' + y' = 1 - ye^{xy}$$

so

$$y' = \frac{1 - ye^{xy}}{1 + xe^{xy}}.$$ Dividing by e^{xy} gives $$y' = \frac{e^{-xy} - y}{e^{-xy} + x}$$

2. $\boxed{6}$ Verify that $\phi(x) = \dfrac{2}{1 - ce^x}$, where c is an arbitrary constant, is a one-parameter family of solutions to

$$\frac{dy}{dx} = \frac{y(y-2)}{2}.$$

$$\varphi' = \frac{2ce^x}{(1-ce^x)^2}$$

$$\frac{\varphi(\varphi-2)}{2} = \frac{2}{(1-ce^x)}\left(\frac{2}{1-ce^x} - 2\right)\frac{1}{2}$$

$$= \frac{1}{1-ce^x}\left[\frac{2}{1-ce^x} - \frac{2 - 2ce^x}{1-ce^x}\right]$$

$$= \frac{2ce^x}{(1-ce^x)^2}$$

So $\varphi' = \dfrac{\varphi(\varphi-2)}{2}$ as desired.