Math 274 | Quiz 10 | Name: ________________

17 July 2017 | Show Appropriate Work | Scaled to $\boxed{10}$.

1. $\boxed{6}$ Find the general solution for the following.

(a) $y'' - 4y = 0$ $\quad r^2 - 4 = (r-2)(r+2) = 0$

$y = C_1 e^{2t} + C_2 e^{-2t}$

(d) $t^2y'' + ty' - y = 0$ $\quad r^2 - 1 = (r-1)(r+1) = 0$

$y = C_1 t + C_2 t^{-1}$

(b) $4y'' - 4y' + y = 0$ $\quad 4r^2 - 4r + 1 = (2r-1)^2 = 0$

$y = C_1 e^{t/2} + C_2 t e^{t/2}$

(e) $4t^2y'' + y = 0$ $\quad 4r^2 - 4r + 1 = 0$

$y = C_1 \sqrt{t} + C_2 \sqrt{t} \ln t$

(c) $y''' + 2y'' + 5y' = 0$ $\quad r^2 + 2r + 5 = 0$

$r = \dfrac{-2 \pm \sqrt{4-20}}{2}$

$r = -1 \pm 2i$

$y = C_1 e^{-t} \cos 2t + C_2 e^{-t} \sin 2t + C_3$

(f) $t^2y'' + 3ty' + 5y = 0$

$y = C_1 t^{-1} \cos(2 \ln t) + C_2 t^{-1} \sin(2 \ln t)$

2. $\boxed{4}$ Find the general solution for

$$y'' - 4y = (16t - 8)e^{-2t}.$$

$y_p = t(At + B)e^{-2t} = (At^2 + Bt)e^{-2t}$

$y_p' = [2At + B - 2At^2 - 2Bt]e^{-2t}$

$y_p'' = [2A - 4At - 2B - 4At - 2B + 4At^2 + 4Bt]e^{-2t}$

$y_p'' - 4y_p = [4At^2 + (-8A + 4B)t + (2A - 4B)]e^{-2t} + [-4At^2 - 4Bt]e^{-2t}$

$t^2e^{-2t}: \quad 4A - 4A = 0$

$te^{-2t}: \quad -8A + 4B - 4B = 16 \quad \Rightarrow A = -2$

$e^{-2t}: \quad 2A - 4B = -8 \quad \Rightarrow B = 1$

so $y = C_1 e^{2t} + C_2 e^{-2t} + (-2t^2 + 1t)e^{-2t}$ is the general solution.

3. $\boxed{4}$ Use Reduction of Order and the given solution to find a second linearly independent solution to the following equation.

$$ty'' + (1-2t)y' + (t-1)y = 0, \quad t > 0, \quad y_1 = e^t$$

$$p(t) = \frac{1-2t}{t} = \frac{1}{t} - 2 \qquad e^{-\int p(t)\,dt} = e^{-(\ln t - 2t)} = t^{-1} \cdot e^{2t} = \frac{e^{2t}}{t}$$

$$\text{so} \quad y_2 = y_1 \int \frac{\frac{e^{2t}}{t}}{e^{2t}}\,dt = y_1 \int \frac{dt}{t} = e^t \cdot \ln t$$

4. $\boxed{6}$ Solve the inital value problem

$$t^2y'' + ty' - y = t^2e^t, \quad y(1) = 0, \quad y'(1) = e.$$

$$y_1 = t \qquad W[t, t^{-1}] = \begin{vmatrix} t & t^{-1} \\ 1 & -t^{-2} \end{vmatrix} = -t^{-1} - t^{-1} = -2t^{-1}$$
$$y_2 = t^{-1}$$

$$V_1 = \int \frac{-e^t t^{-1}}{-2t^{-1}}\,dt = \frac{1}{2}\int e^t\,dt = \frac{1}{2}e^t$$

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$$V_2 = \int \frac{e^t t}{-2t^{-1}}\,dt = -\frac{1}{2}\int t^2 e^t\,dt = -\frac{1}{2}\left[t^2e^t - 2te^t + 2e^t\right]$$

$$\text{so } y = C_1 t + C_2 t^{-1} + \frac{1}{2}te^t - \frac{1}{2}\left[te^t - 2e^t + \frac{2e^t}{t}\right]$$

$$= C_1 t + C_2 t^{-1} + e^t\left(1 - \frac{1}{t}\right)$$

$$y' = C_1 - \frac{C_2}{t^2} + e^t\left(1 - \frac{1}{t} + \frac{1}{t^2}\right)$$

$$y = e^t(1 - 1/t)$$

$y(1) = 0$

so $C_1 + C_2 = 0$

$y'(1) =$
$C_1 - C_2 + e = e$

so
$C_1 - C_2 = 0$

$$\left.\begin{array}{l} C_1 + C_2 = 0 \\ C_1 - C_2 = 0 \end{array}\right\} C_1 = C_2 = 0$$