

1. 4 Find the general solution for the following, assume $t > 0$.

(a) $t^2 y'' - t y' + y = 0$

$$r^2 - 2r + 1 = 0$$

$$(r-1)^2 = 0 \quad r=1$$

$$y = C_1 t + C_2 t \ln t$$

(b) $t^2 y'' + t y' + 4y = 0$

$$r^2 + 4 = 0$$

$$r = \pm 2i$$

$$y = C_1 \cos(2 \ln t) + C_2 \sin(2 \ln t)$$

2. 2 Solve the initial value problem

$$t^2 y'' - 6y = 0, \quad y(1) = 1, \quad y'(1) = 8.$$

$$r^2 - r - 6 = (r-3)(r+2) = 0$$

$$r=3, \quad r=-2$$

$$y = C_1 t^3 + C_2 t^{-2}$$

$$y' = 3C_1 t^2 - 2C_2 t^{-3}$$

$$y(1) = C_1 + C_2 = 1$$

$$y'(1) = 3C_1 - 2C_2 = 8$$

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$$3C_1 - 2C_2 = 8$$

$$2C_1 + 2C_2 = 2$$

$$5C_1 = 10 \quad \text{so} \quad C_1 = 2$$

$$C_2 = -1$$

$$\text{so } y = 2t^3 - t^{-2}$$

3. 4 Use the table provided to compute the following Laplace transforms.

(a) $\mathcal{L}\{e^{2t} \sin 3t + 6t^4\}$

$$= \frac{3}{(s-2)^2 + 9} + 6 \cdot \frac{4!}{s^5}$$

(b) $\mathcal{L}\{8 - 2e^{-t} \cos 7t\}$

$$= \frac{8}{s} - 2 \cdot \frac{s+1}{(s+1)^2 + 49}$$