Math 274 — Quiz 12b — Name: ____________________

19 July 2017 — Show Appropriate Work — Scaled to $\boxed{10}$.

1. Compute the inverse Laplace Transform for the following.

(a) $\boxed{1}$ $$F(s) = \frac{3}{s+4}$$

$$f = 3e^{-4t}$$

(b) $\boxed{1}$ $$G(s) = \frac{3}{2s+4} = \frac{3/2}{s+2}$$

$$g = \frac{3}{2} e^{-2t}$$

(c) $\boxed{1}$ $$H(s) = \frac{3}{(s+4)^4} = \frac{1}{2} \cdot \frac{3!}{(s+4)^4}$$

$$h = \frac{1}{2} e^{-4t} t^3$$

(d) $\boxed{1}$ $$J(s) = \frac{3}{s^2+4} = \frac{3}{2} \cdot \frac{2}{s^2+4}$$

$$j = \frac{3}{2} \sin 2t$$

(e) $\boxed{1}$ $$K(s) = \frac{4s}{s^2+6} =$$

$$k = 4 \cos\left(\sqrt{6}\, t\right)$$

(f) $\boxed{3}$ $$M(s) = \frac{3s}{s^2+2s+5} = \frac{3(s+1) - \frac{3}{2}\cdot 2}{(s+1)^2 + 2^2}$$

$$3e^{-t}\cos 2t - \frac{3}{2} e^{-t} \sin 2t$$

(g) $\boxed{2}$ $\quad F(s) = \ln\left(\dfrac{s^2-1}{s^2+1}\right)$ $\quad$ [SEE # 33-36 IN YOUR TEXT.]

$$= \ln(s+1) + \ln(s-1) - \ln(s^2+1)$$

$$F'(s) = \frac{1}{s+1} + \frac{1}{s-1} - \frac{2s}{s^2+1}$$

Since $\mathcal{L}^{-1}\{F'\} = -t\,f(t)$

$$f(t) = -\frac{1}{t}\left(e^{-t} + e^{t} - 2\cos t\right)$$

(h) $\boxed{4}$ $\quad G(s) = \dfrac{4s^2+3s+4}{(s+1)(s^2+4s+8)} = \dfrac{1}{s+1} + \dfrac{3s-4}{(s+2)^2+2^2} = \dfrac{1}{s+1} + \dfrac{3(s+2)-10}{(s+2)^2+2^2}$

$$\frac{4s^2+3s+4}{(s+1)(s^2+4s+8)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+4s+8}$$

$$g(t) = e^{-t} + 3e^{-2t}\cos 2t - 5e^{-2t}\sin 2t$$

$$4s^2+3s+4 = A(s^2+4s+8) + (Bs+C)(s+1)$$

Let $s=-1$: $\quad 5 = A(5)$ so $A=1$

Eq. coeff

s^2: $\quad 4 = A+B$ so $B=3$

s^0: $\quad 4 = 8A+C$ so $C=-4$

(i) $\cancel{\boxed{2}}$ $\boxed{6}$ $\quad \left[s^2Y(s) - 2s - 12\right] - 2\left[sY(s) - 2\right] + 5Y(s) = \dfrac{-8}{s+1}$

$$Y(s)\left[s^2-2s+5\right] = \frac{-8}{s+1} + 2s + 8 = \frac{-8 + 2s^2 + 10s + 8}{s+1} = \frac{2s^2+10s}{s+1}$$

$$Y(s) = \frac{2s^2+10s}{(s+1)(s^2-2s+5)}$$

$$\frac{2s^2+10s}{(s+1)(s^2-2s+5)} = \frac{A}{s+1} + \frac{Bs+C}{s^2-2s+5}$$

$$2s^2+10s = A(s^2-2s+5) + (Bs+C)(s+1)$$

$s=-1$: $\quad -8 = A(8)$ so $A=-1$

Eq. Coeff

s^2: $\quad 2 = A+B$ so $B=3$

s^0: $\quad 0 = 5A+C$ so $C=5$

so $Y(s) = \dfrac{-1}{s+1} + \dfrac{3s+5}{(s-1)^2+2^2}$

$$= \frac{-1}{s+1} + \frac{3(s-1)}{(s-1)^2+2^2} + \frac{4\cdot 2}{(s-1)^2+2^2}$$

Finally,

$$y(t) = 3e^{t}\cos 2t + 4e^{t}\sin 2t - e^{-t}$$