Math 274 | Quiz 13b | Name: ____________
Due: 25 July 2017 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{5}$ Use the method of Laplace Transforms to solve the initial value problem.

$$4y'' + 2ty - 10y = 4t^3, \quad y(0) = y'(0) = 0$$

$4s^2Y - 2(Y + sY') - 10Y = \frac{24}{s^4}$ which is 1st order linear,

$-2sY' + (4s^2 - 12)Y = \frac{24}{s^4}$

$Y' + \left(\frac{6}{s} - 2s\right)Y = \frac{-12}{s^5}$ is standard form.

$\mu(s) = e^{\int(\frac{6}{s} - 2s)\,ds} = s^6 e^{-s^2}$ is the integrating factor.

The solution is then

$Y = \frac{e^{s^2}}{s^6}\int\left(-12/s^5 \cdot s^6 e^{-s^2}\right)ds = \frac{e^{s^2}}{s^6}\left(6e^{-s^2} + C\right) = \frac{6}{s^6} + \frac{Ce^{s^2}}{s^6}$.

We know $Y \xrightarrow[s\to\infty]{} 0$, so $C = 0$.

$Y(s) = \frac{6}{s^6}$

so

$y(t) = \frac{6}{5!}t^5 = \frac{t^5}{20}$ solves the original I.V.P.