

1. 3 Express the following using unit step functions.

$$f(t) = \begin{cases} e^{3t}, & 0 < t < 3 \\ \cos 2t, & 3 < t < 5 \\ 3, & t > 5 \end{cases}$$

$$f(t) = e^{3t} + u(t-3)(\cos 2t - e^{3t}) + u(t-5)(3 - \cos 2t)$$

2. 3 Compute the Laplace transform of the following function.

$$f(t) = e^{2t} + u(t-2)(t^2 - e^{2t})$$

$$F(s) = \frac{1}{s-2} + e^{-2s} \mathcal{L}\{(t+2)^2 - e^{2t+4}\}$$

$$= \frac{1}{s-2} + e^{-2s} \left[\frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s} - e^4 \cdot \frac{1}{s-2} \right]$$

3. 4 Compute the inverse Laplace transform of the following.

$$F(s) = \frac{e^{-2s}(3s^2 + s + 8)}{(s+1)(s^2+4)}$$

$$\frac{3s^2 + s + 8}{(s+1)(s^2+4)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+4}$$

$$3s^2 + s + 8 = A(s^2+4) + (Bs+C)(s+1)$$

$$s=-1 : 10 = 5A \quad \text{so } A=2$$

$$s^2 : 3 = A+B \quad \text{so } B=1$$

$$s^0 : 8 = 4A+C \quad \text{so } C=0$$

$$F(s) = e^{-2s} \left[\frac{2}{s+1} + \frac{s}{s^2+4} \right]$$

$$= u(t-2) \left[2e^{-(t-2)} + \cos(2(t-2)) \right]$$