Math 274 | Quiz 15 | Name: ______

26 July 2017 | Show Appropriate Work | Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{10}$ Use the method of Laplace transforms to solve the inital value problem.

$$y'' + y = f(t); \qquad f(t) = \begin{cases} 2e^t, & 0 < t < 2 \\ 0, & 2 < t \end{cases} \qquad y(0) = 0,\ y'(0) = 0$$

$$f(t) = 2e^t + u(t-2)(-2e^t)$$

$$s^2Y + Y = \frac{2}{s-1} + e^{-2s}\mathcal{L}\{-2e^{t+2}\}$$

$$Y(s^2+1) = \frac{2}{s-1} - e^{2-2s}\left(\frac{2}{s-1}\right)$$

$$Y = \frac{2}{(s-1)(s^2+1)} - e^{2-2s}\left(\frac{2}{(s-1)(s^2+1)}\right).$$

we do a quick partial fraction decomposition

$$\frac{2}{(s-1)(s^2+1)} = \frac{A}{s-1} + \frac{Bs+C}{s^2+1}$$

$$2 = A(s^2+1) + (Bs+C)(s-1)$$

$s=1$: $A=1$

s^2 : $A+B=0$ so $B=-1$

s^0 : $A-C=2$ so $C=-1$,

so

$$Y = \left(\frac{1}{s-1} - \frac{s}{s^2+1} - \frac{1}{s^2+1}\right) - e^{2-2s}\left(\frac{1}{s-1} - \frac{s}{s^2+1} - \frac{1}{s^2+1}\right)$$

$$y = e^t - \cos t - \sin t - e^2 u(t-2)\left[e^{t-2} - \cos(t-2) - \sin(t-2)\right]$$