

1. 10 Consider the system  $\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t)$  with  $\mathbf{A} = \begin{bmatrix} 2 & 2 \\ 4 & 0 \end{bmatrix}$ . Find a fundamental matrix for the system.

$$\begin{vmatrix} 2-r & 2 \\ 4 & -r \end{vmatrix} = (2-r)(-r) - 8 \\ = r^2 - 2r - 8 \\ = (r-4)(r+2)$$

$$r_1 = 4 \quad \begin{bmatrix} -2 & 2 \\ 4 & -4 \end{bmatrix} \vec{u}_1 = \vec{0} \quad \text{so} \quad \vec{u}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$r_2 = -2 \quad \begin{bmatrix} 4 & 2 \\ 4 & 2 \end{bmatrix} \vec{u}_2 = \vec{0} \quad \text{so} \quad \vec{u}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\underline{\mathbf{X}} = \begin{bmatrix} e^{4t} & e^{-2t} \\ e^{4t} & -2e^{-2t} \end{bmatrix}$$