Math 274 Quiz 17 Name: ________________

31 July 2017 Show Appropriate Work Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{4}$ Consider the system $\mathbf{x}'(t) = \begin{bmatrix} 2 & -9 \\ 1 & 2 \end{bmatrix} \mathbf{x}(t)$. Find a fundamental solution set for the system.

$|A - rI| = (2-r)^2 + 9 = 0$ so $r - 2 = \pm 3i$

$r = 2 \pm 3i$

We will use $r = 2+3i$ as usual. Now find the eigenvector associated

$$\begin{bmatrix} -3i & -9 \\ 1 & -3i \end{bmatrix} \vec{u} = \vec{0} \qquad \text{we choose } \vec{u} = \begin{bmatrix} 3i \\ 1 \end{bmatrix}$$

so $r = 2+3i$ $(\alpha = 2, \beta = 3)$

& $\vec{u} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + i \begin{bmatrix} 3 \\ 0 \end{bmatrix}$ $\left(\vec{a} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \vec{b} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} \right)$

so a fund solution set is

$$\left\{ e^{2t}\left(\cos 3t \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \sin 3t \begin{bmatrix} 3 \\ 0 \end{bmatrix} \right), e^{2t}\left(\sin 3t \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \cos 3t \begin{bmatrix} 3 \\ 0 \end{bmatrix} \right) \right\}$$

<u>Note</u>: $\vec{u} = \begin{bmatrix} 3 \\ -i \end{bmatrix}$ also works, as does $\vec{u} = \begin{bmatrix} -3 \\ i \end{bmatrix}$

2. $\boxed{6}$ Use variation of parameters to find a particular solution for $\mathbf{x}'(t) = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 25e^{3t} \\ 30 \end{bmatrix}$.

Recall, if $\mathbf{U} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible, then $\mathbf{U}^{-1} = \dfrac{1}{|\mathbf{U}|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

find $\underline{X}$

$$\begin{vmatrix} 2-r & 2 \\ 2 & -1-r \end{vmatrix} = (2-r)(-1-r) - 4 = r^2 - r - 6 = 0 \quad \text{so} \quad r_1 = 3, \ r_2 = -2$$

$$r_1: \begin{bmatrix} -1 & 2 \\ 2 & -4 \end{bmatrix} \vec{u}_1 = \vec{0} \quad \text{so} \quad \vec{u}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$r_2: \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} \vec{u}_2 = \vec{0} \quad \text{so} \quad \vec{u}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\text{so} \quad \underline{X} = \begin{bmatrix} 2e^{3t} & e^{-2t} \\ e^{3t} & -2e^{-2t} \end{bmatrix}$$

$$|\underline{X}| = -4e^t - e^t = -5e^t \qquad \underline{X}^{-1} = \frac{1}{-5e^t} \begin{bmatrix} -2e^{-2t} & -e^{-2t} \\ -e^{3t} & 2e^{3t} \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 2e^{-3t} & e^{-3t} \\ e^{2t} & -2e^{2t} \end{bmatrix}$$

$$\underline{X}^{-1} f = \frac{1}{5} \begin{bmatrix} 2e^{-3t} & e^{-3t} \\ e^{2t} & -2e^{2t} \end{bmatrix} \begin{bmatrix} 25e^{3t} \\ 30 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 50 + 30e^{-3t} \\ 25e^{5t} - 60e^{2t} \end{bmatrix} = \begin{bmatrix} 10 + 6e^{-3t} \\ 5e^{5t} - 12e^{2t} \end{bmatrix}$$

$$\int \underline{X}^{-1} f = \begin{bmatrix} 10t - 2e^{-3t} \\ e^{5t} - 6e^{2t} \end{bmatrix}$$

$$\underline{X} \int \underline{X}^{-1} f = \begin{bmatrix} 2e^{3t} & e^{-2t} \\ e^{3t} & -2e^{-2t} \end{bmatrix} \begin{bmatrix} 10t - 2e^{-3t} \\ e^{5t} - 6e^{2t} \end{bmatrix} = \begin{bmatrix} 20te^{3t} - 4 + e^{3t} - 6 \\ 10te^{3t} - 2 - 2e^{3t} + 12 \end{bmatrix}$$

so

$$\vec{x}_p = te^{3t} \begin{bmatrix} 20 \\ 10 \end{bmatrix} + e^{3t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + \begin{bmatrix} -10 \\ 10 \end{bmatrix}$$