Math 274 Quiz 19 Name: ____________

2 Aug 2017 Show Appropriate Work Point Values in $\boxed{\text{boxes}}$.

1. $\boxed{2}$ Consider the system $\mathrm{x}'(t) = \begin{bmatrix} 3 & -4 \\ 1 & 3 \end{bmatrix} \mathrm{x}(t)$. Find a general solution for the system.

$$\begin{vmatrix} 3-r & -4 \\ 1 & 3-r \end{vmatrix} = (3-r)^2 + 4 = 0 \quad \text{so} \quad r = 3 \pm 2i$$

For $r = 3+2i$

$$\begin{bmatrix} -2i & -4 \\ 1 & -2i \end{bmatrix} \vec{u}_1 = \vec{0} \quad \text{so} \quad \vec{u}_1 = \begin{bmatrix} 2i \\ 1 \end{bmatrix} \text{ is one eigenvector}$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix} + i \begin{bmatrix} 2 \\ 0 \end{bmatrix}.$$

$$\vec{x} = C_1 e^{3t} \left(\cos 2t \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \sin 2t \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right) + C_2 e^{3t} \left(\sin 2t \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \cos 2t \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right)$$

2. $\boxed{2}$ Write a system in normal form with initial data for the mixing problem on the board.

Something of the form

$$\vec{x}' = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \vec{x} + \vec{f} \quad , \quad \vec{x}(0) = \vec{x}_0$$

3. $\boxed{3}$ In class on Tuesday we found that if $\mathbf{X}(t)$ was a a fundamental matrix for $\mathbf{x}' = \mathbf{A}\mathbf{x}(t)$ then $e^{\mathbf{A}t} = \mathbf{X}(t)\mathbf{X}(0)^{-1}$. Use this method to compute $e^{\mathbf{A}t}$ for $\mathbf{A} = \begin{bmatrix} 3 & 2 \\ 5 & 6 \end{bmatrix}$.

$$\begin{vmatrix} 3-r & 2 \\ 5 & 6-r \end{vmatrix} = (3-r)(6-r) - 10 = r^2 - 9r + 8 = (r-1)(r-8)$$

$$r=1: \quad \begin{bmatrix} 2 & 2 \\ 5 & 5 \end{bmatrix}, \quad \vec{u}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad r=8: \quad \begin{bmatrix} -5 & 2 \\ 5 & -2 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$\underline{X} = \begin{bmatrix} e^t & 2e^{8t} \\ -e^t & 5e^{8t} \end{bmatrix} \qquad \underline{X}(0) = \begin{bmatrix} 1 & 2 \\ -1 & 5 \end{bmatrix} \quad \text{so} \quad \underline{X}(0)^{-1} = \frac{1}{7}\begin{bmatrix} 5 & -2 \\ 1 & 1 \end{bmatrix}$$

$$\text{so} \quad e^{At} = \frac{1}{7}\begin{bmatrix} e^t & 2e^{8t} \\ -e^t & 5e^{8t} \end{bmatrix}\begin{bmatrix} 5 & -2 \\ 1 & 1 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 5e^t + 2e^{8t} & -2e^t + 2e^{8t} \\ -5e^t + 5e^{8t} & 2e^t + 5e^{8t} \end{bmatrix}$$

4. $\boxed{3}$ We also found that if the characteristic equation for $\mathbf{x}' = \mathbf{A}\mathbf{x}(t)$ has a double root r then $e^{\mathbf{A}t} = e^{rt}\left(\mathbf{I} + (\mathbf{A} - r\mathbf{I})t\right)$. Use this method to compute $e^{\mathbf{A}t}$ for $\mathbf{A} = \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}$.

$$\begin{vmatrix} 3-r & 1 \\ -1 & 1-r \end{vmatrix} = (3-r)(1-r) + 1 = r^2 - 4r + 4 = (r-2)^2$$

$$e^{At} = e^{2t}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}t\right) = e^{2t}\begin{bmatrix} 1+t & t \\ -t & 1-t \end{bmatrix}$$