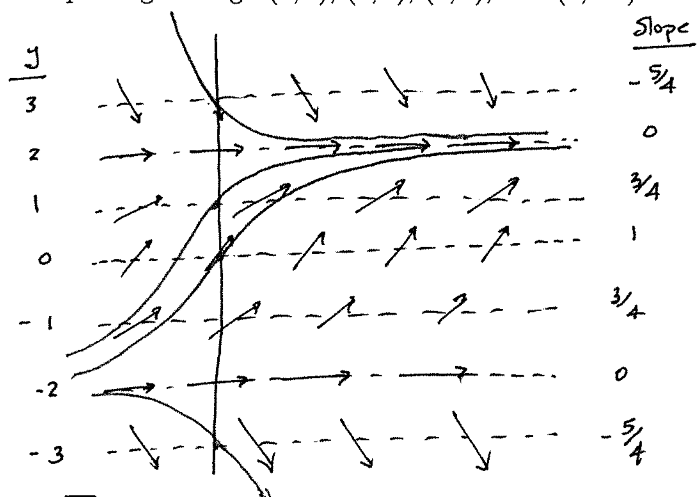
Math 274 — Quiz 2 — Name: ______________

28 June 2017 — Show Appropriate Work — Scaled to $\boxed{10}$.

1. Consider the ODE
$$\frac{dy}{dx} = \frac{4-y^2}{4}.$$

(a) $\boxed{4}$ The equation is autonomous, i.e. there is no explicit dependence on x on the right side, so isoclines are horizontal lines. Sketch the direction field including isoclines, with direction arrows, corresponding to $y = -3, -2, -1, 0, 1, 2,$ and 3. Sketch solution curves passing through $(0,0)$, $(0,1)$, $(0,3)$, and $(0,-3)$.



(b) $\boxed{4}$ The equation is separable, find the explicit general solution, i.e. find the explicit solution containing an arbitrary constant c.

$$\frac{dy}{dx} = \frac{4-y^2}{4}$$

$$\int \frac{4}{4-y^2}\,dy = \int dx$$

Note: $\frac{4}{4-y^2} = \frac{A}{2-y} + \frac{B}{2+y}$

$$4 = A(2+y) + B(2-y)$$

Let $y=2$, $A=1$

Let $y=-2$, $B=1$

so $\int \left(\frac{1}{2+y} + \frac{1}{2-y} \right) dy = \int dx$

$$\ln|2+y| - \ln|2-y| = x + C$$

$$\ln\left|\frac{2+y}{2-y}\right| = x + C$$

$$\frac{2+y}{2-y} = Ce^x$$

so $y = \dfrac{2Ce^x - 2}{1 + Ce^x}$

(c) $\boxed{2}$ Find the solution passing through $(0,0)$.

$0 = \dfrac{2C-2}{1+C}$ so $C=1$ giving $y = \dfrac{2e^x-2}{1+e^x}$.

2. Consider the ODE

$$\frac{dx}{dt} = \frac{x+2}{t^2}.$$

(a) $\boxed{4}$ The equation is separable, use those methods to find the explicit general solution.

$$\int \frac{dx}{x+2} = \int \frac{dt}{t^2}$$

$$\ln|x+2| = -\frac{1}{t} + C$$

$$x+2 = Ce^{-1/t}$$

$$x = Ce^{-1/t} - 2$$

(b) $\boxed{4}$ The equation is linear, use those methods to find the explicit general solution.

$$\frac{dx}{dt} - \frac{1}{t^2}x = \frac{2}{t^2} \qquad \mu(t) = e^{\int -\frac{1}{t^2}dt} = e^{1/t}$$

$$x = e^{-1/t}\int e^{1/t}\cdot\frac{2}{t^2}\,dt \qquad u = \frac{1}{t} \qquad du = -\frac{1}{t^2}\,dt$$

$$= e^{-1/t}\int(-2e^u)\,du = e^{-1/t}\left[-2e^{1/t} + c\right]$$

$$= Ce^{-1/t} - 2$$

(c) $\boxed{1}$ Solve the IVP ~~$x(0) = 2$.~~ $x(1) = 2$

$Ce^{-1} - 2 = 2$ so $C = 4e$ so $x(t) = 4e^{1-1/t} - 2$

(d) $\boxed{1}$ Solve the IVP $x(2) = 0$.

$Ce^{-1/2} - 2 = 0$ so $C = 2e^{1/2}$ giving $x(t) = 2e^{\frac{1}{2}-\frac{1}{t}} - 2$.