We have discussed three types of first order differential equations; separable, linear, and exact. There is exactly one of each below.

1. [4] Find the explicit general solution of
   \[ \frac{dx}{dt} + x = t. \]

   \[ x(t) = \frac{1}{t} \left( t^3 \right) + Ce^{-\frac{1}{t}} \]

   We will use \( M(t) = t \).

2. [3] Find the explicit general solution of
   \[ \frac{dy}{dx} = y^2x. \]

   \[ \int \frac{dy}{y^2} = \int x \, dx \]

   \[ -\frac{1}{y} = \frac{x^2}{2} + C \]

   \[ y = \frac{2}{C - x^2} \]

3. [3] Find the general solution of
   \[ \frac{dy}{dx} = \frac{y - \sec^2 x}{ey - x}. \]

   \[ (e^{y-x}) \, dy = (y - \sec^2 x) \, dx \]

   \[ \sec^2 x - y \, dx + (e^{y-x}) \, dy = 0 \]

   \[ \frac{dy}{dx} (\sec^2 x - y) = -1 \]

   \[ \frac{dy}{dx} (e^{y-x}) = -1 \]

   \[ e^{y-x} \]

   \[ t_2 - x - xy + e^y = C \]