

We have discussed three types of first order differential equations; separable, linear, and exact. There is exactly one of each below.

1. 4 Find the explicit general solution of

$$t \frac{dx}{dt} = t^2 - x.$$

$$t \frac{dx}{dt} + x = t^2$$

$$\frac{dx}{dt} + \frac{1}{t}x = t$$

$$u(t) = e^{\int \frac{1}{t} dt} = e^{\ln|t|} = |t|$$

we will use  $u(t) = t$ .

$$\begin{aligned} x(t) &= \frac{1}{t} \int t^2 dt = \frac{1}{t} \left( \frac{t^3}{3} + C \right) \\ &= \frac{t^2}{3} + \frac{C}{t} \end{aligned}$$

2. 3 Find the explicit general solution of

$$\frac{dy}{dx} = y^2 x.$$

$$\int \frac{dy}{y^2} = \int x dx$$

$$-\frac{1}{y} = \frac{x^2}{2} + C$$

$$y = \frac{1}{C - \frac{x^2}{2}} = \frac{2}{C - x^2}$$

3. 3 Find the general solution of

$$\frac{dy}{dx} = \frac{y - \sec^2 x}{e^y - x}.$$

$$(e^y - x) dy = (y - \sec^2 x) dx$$

$$(\sec^2 x - y) dx + (e^y - x) dy = 0$$

$$\left. \begin{aligned} \frac{d}{dy} (\sec^2 x - y) &= -1 \\ \frac{d}{dx} (e^y - x) &= -1 \end{aligned} \right\} \begin{array}{l} \text{so} \\ \text{exact} \end{array}$$

$$F(x, y) = \tan x - xy + e^y, \quad \text{so}$$

$$\tan x - xy + e^y = C$$