

1. Use appropriate substitutions to find the explicit general solution for the following.

(a) 4 $\frac{dy}{dx} + \frac{y}{2} = \frac{x}{y}$ so $y \frac{dy}{dx} + \frac{1}{2} y^2 = x$ Bernoulli:

Let $v = y^2$ so $\frac{dv}{dx} = 2y \frac{dy}{dx}$.

$\frac{1}{2} \frac{dv}{dx} + \frac{1}{2} v = x$

$\frac{dv}{dx} + v = 2x$ $\mu(x) = e^{\int dx} = e^x$

so $v = e^{-x} \int 2x e^x dx = e^{-x} [2x e^x - 2e^x + c] = 2x - 2 + c e^{-x}$

$u = 2x$ $dv = e^x dx$
 $du = 2dx$ $v = e^x$

Finally, $y = \pm \sqrt{2x - 2 + c e^{-x}}$

(b) 4 $\frac{dy}{dx} = \sqrt{2x + y} - 2$ Let $v = 2x + y$
so $\frac{dv}{dx} = 2 + \frac{dy}{dx}$.

$\frac{dv}{dx} = \sqrt{v}$

$\int \frac{dv}{\sqrt{v}} = \int dx$

so $2\sqrt{v} = x + c$

$v = \left(\frac{x+c}{2}\right)^2$

$y = \left(\frac{x+c}{2}\right)^2 - 2x$

2. In one of the two equations above it is likely that you lost a solution. For which equation did you lose a solution and what is the lost solution?

in (b) we lost the solution $v = 0$, i.e. $2x + y = 0$

OR $y = -2x$