Math 274 | Quiz 7 | Name: ______________
11 July 2017 | Show Appropriate Work | Point Values in [boxes].

Method of Undetermined Coefficients

To find a particular solution to

$$ay'' + by' + cy = P_m(t)e^{rt}$$

where $P_m(t)$ is a polynomial of degree m, use the form

$$y_p(t) = t^s\left(A_m t^m + \cdots + A_1 t + A_0\right)e^{rt};$$

if r is not a root of the associated auxiliary equation, take $s = 0$; if r is a simple root, take $s = 1$; and if r is a double root, take $s = 2$.

To find a particular solution to

$$ay'' + by' + cy = P_m(t)e^{\alpha t}\cos\beta t + Q_n(t)e^{\alpha t}\sin\beta t$$

where $P_m(t)$ and $Q_n(t)$ are polynomials of degree m and n, respectively, use the form

$$y_p(t) = t^s\left(A_k t^k + \cdots + A_1 t + A_0\right)e^{\alpha t}\cos\beta t + t^s\left(B_k t^k + \cdots + B_1 t + B_0\right)e^{\alpha t}\sin\beta t;$$

where k is the larger of m and n. If $\alpha + i\beta$ is not a root of the associated auxiliary equation, take $s = 0$; if so take $s = 1$.

Use the Method of Undetermined Coefficients to find a particuar solution for the following.

1. [2] $y'' - 8y = (t+8)e^{3t}$ $\qquad y_p = (t+2)e^{3t}$

$y_p = (At+B)e^{3t}$

$y_p' = (A + 3At + 3B)e^{3t}$

$y_p'' = (3A + 3A + 9At + 9B)e^{3t}$

so $y_p = (t+2)e^{3t}$

$y_p'' - 8y_p = [6A + 9At + 9B]e^{3t} - [8At + 8B]e^{3t} = (t+8)e^{3t}$

so $9A - 8A = 1 \Rightarrow A = 1$ $\quad 6A + 9B - 8B = 8 \Rightarrow B = 2$

2. [2] $y'' + 3y' + 12y = -50\sin 2t$

try $y_p = A\cos 2t + B\sin 2t$

$y_p' = -2A\sin 2t + 2B\cos 2t$

$y_p'' = -4A\cos 2t - 4B\sin 2t$

so $[-4A\cos 2t - 4B\sin 2t]$

$+ [-6A\sin 2t + 6B\cos 2t]$

$+ [12A\cos 2t + 12B\sin 2t] = -50\sin 2t$

$\cos 2t$: $-4A + 6B + 12A = 0$ $\qquad 8A + 6B = 0$, $\quad 4A + 3B = 0$

$\sin 2t$: $-4B - 6A + 12B = 0$ $\qquad$ ~~4B-3A=-50~~ $\quad 8B - 6A = -50$

so $12A + 9B = 0$

$-12A + 16B = -100$

$25B = -100$ $\quad$ so $B = -4$, $A = 3$

so $y_p = 3\cos 2t - 4\sin 2t$

3. [3] $y'' - y' - 6y = (40t + 23)e^{3t} - 12t + 4$

$y_p = t(At + B)e^{3t} + (Ct + D)$

$$\boxed{y_p = (4t^2 + 3t)e^{3t} + (2t - 1)}$$

$y_p' = (2At + B + 3At^2 + 3Bt)e^{3t} + C$

$y_p'' = (2A + 6At + 3B + 6At + 3B + 9At^2 + 9Bt)e^{3t}$

so

$[9At^2 + (12A + 9B)t + 2A + 6B]e^{3t} + [-3At^2 - (2A + 3B)t - B]e^{3t} + [-6At^2 - 6Bt]e^{3t} - C - 6Ct - 6D$

$= (40t + 23)e^{3t} - 12t + 4$

t: $-6C = -12 \Rightarrow C = 2$

t^0: $-C - 6D = 4 \Rightarrow D = -1$

te^{3t}: $12A + 9B - 2A - 3B - 6B = 40$ so $10A = 40$ so $A = 4$

e^{3t}: $2A + 6B - B = 23$ so $8 + 5B = 23$ so $B = 3$

4. [3] $y'' - y' - 6y = -250t\cos t$

Try $y_p = (At + B)\cos t + (Ct + D)\sin t$

so $y_p = (35t - 2)\cos t + (5t - 11)\sin t$

$y_p' = A\cos t - (At + B)\sin t + C\sin t + (Ct + D)\cos t$

$= [Ct + D + A]\cos t + [C - At - B]\sin t$

$y_p'' = C\cos t - [Ct + D + A]\sin t - A\sin t + [C - At - B]\cos t$

$= [2C - At - B]\cos t + [-Ct - D - 2A]\sin t$

so $[2C - At - B]\cos t + [-Ct - D - 2A]\sin t + [-Ct - D - A]\cos t + [-C + At + B]\sin t + [-6At - 6B]\cos t + [-6Ct - 6D]\sin t$

$= -250t\cos t$

$t\cos t$: $-A - C - 6A = -250$ so $7A + C = 250$

$t\sin t$: $-C + A - 6C = 0$ so $7C - A = 0 \Rightarrow 49C - 7A = 0$ } so $C = 250$, so $C = 5$, $A = 35$

$\cos t$: $2C - B - D - A - 6B = 0$ so $10 - 7B - D - 35 = 0 \Rightarrow 7B + D = -25$

$\sin t$: $-D - 2A - C + B - 6D = 0$ so $-D - 70 - 5 + B - 6D = 0$ $B - 7D = 75$

$49B + 7D = -175$
$B - 7D = 75$ } $50B = -100$ so $B = -2$, $D = -11$