Method of Undetermined Coefficients
To find a particular solution to
\[ ay'' + by' + cy = P_m(t)e^{rt} \]
where \( P_m(t) \) is a polynomial of degree \( m \), use the form
\[ y_p(t) = t^s (A_m t^m + \cdots + A_1 t + A_0) e^{rt}; \]
if \( r \) is not a root of the associated auxiliary equation, take \( s = 0 \); if \( r \) is a simple root, take \( s = 1 \); and if \( r \) is a double root, take \( s = 2 \).

To find a particular solution to
\[ ay'' + by' + cy = P_m(t)e^{\alpha t} \cos \beta t + Q_n(t)e^{\alpha t} \sin \beta t \]
where \( P_m(t) \) and \( Q_n(t) \) are polynomials of degree \( m \) and \( n \), respectively, use the form
\[ y_p(t) = t^s \left( A_k t^k + \cdots + A_1 t + A_0 \right) e^{\alpha t} \cos \beta t + t^s \left( B_k t^k + \cdots + B_1 t + B_0 \right) e^{\alpha t} \sin \beta t; \]
where \( k \) is the larger of \( m \) and \( n \). If \( \alpha + i\beta \) is not a root of the associated auxiliary equation, take \( s = 0 \); if so take \( s = 1 \).

Use the Method of Undetermined Coefficients to find a particular solution for the following.

1. \[ y'' - 8y = (t + 8)e^{3t} \]

2. \[ y'' + 3y' + 12y = -50 \sin 2t \]
3. \[ y'' - y' - 6y = (40t + 23)e^{3t} - 12t + 4 \]

4. \[ y'' - y' - 6y = -250t \cos t \]