Math 274 Homework **Name:** ______________________

Sections: 4.4,4.5 Scaled to $\boxed{10}$ points.

Due: 22 May 2018

1. Find the appropriate form using the Method of Undetermined Coefficients for a particular solution to the following. **Do not** solve for the unknown constants.

   (a) $\boxed{2}$ $y'' - y' - 6y = e^{3t} + 7t^2$

   $r^2 - r - 6 = 0$

   $r = 3, r = -2$

   $y_p = Ate^{3t} + Bt^2 + Ct + D$

   (b) $\boxed{2}$ $y'' - 4y' + 4y = te^t + 3e^{4t}$

   $(r-2)^2 = 0$

   $y_p = (At + B)e^t + Ce^{4t}$

   (c) $\boxed{2}$ $y'' - 4y' + 4y = t^2 e^{2t}$

   $y_p = t^2 (At^2 + Bt + C)e^{2t}$

   (d) $\boxed{2}$ $y'' - 4y' + 4y = te^{2t} \cos t$

   $y_p = (At + B)e^{2t} \cos t + (Ct + D)e^{2t} \sin t$

2. Find a general solution for the following.

   (a) $\boxed{4}$ $y'' - 3y' + 2y = (12t + 8)e^{-t}$

   $r^2 - 3r + 2 = 0$

   $(r-1)(r-2) = 0$

   $r = 1, r = 2$

   $y = C_1 e^t + C_2 e^{2t}$

   $y = C_1 e^t + C_2 e^{2t} + (2t + 3)e^{-t}$

   $y_p = (At + B)e^{-t}$

   $y_p' = (A - At - B)e^{-t}$

   $y_p'' = (-A - A + At + B)e^{-t}$

   Substituting gives

   $[At - 2A + B]e^{-t} + [-3A + 3At + 3B]e^{-t} + [2At + 2B]e^{-t} = (12t + 8)e^{-t}$

   te^{-t}: $A + 3A + 2A = 12$ so $A = 2$

   e^{-t}: $-2A + B - 3A + 3B + 2B = 8$ so $B = 3$

(b) $\boxed{4}$ $y'' - y' - 6y = (40t+23)e^{3t} - 12t + 4$ $r^2 - r - 6 = 0$, $r = 3$, $r = -2$

$y_p = t(At+B)e^{3t} + (Ct+D)$

$y_p' = (2At + B + 3At^2 + 3Bt)e^{3t} + C$

$y_p'' = (2A + 6At + 3B + 6At + 3B + 9At^2 + 9Bt)e^{3t}$

so

$[9At^2 + (12A+9B)t + 2A + 6B]e^{3t} + [-3At^2 - (2A+3B)t - B]e^{3t} + [-6At^2 - 6Bt]e^{3t} - 6Ct - C - 6D$

$= (40t+23)e^{3t} - 12t + 4$

t: $-6C = -12$ so $C = 2$

t^0: $-C - 6D = 4$ so $D = -1$

te^{3t}: $12A + 9B - 2A - 3B - 6B = 40$ so $A = 4$

e^{3t}: $2A + 6B - B = 23$ so $B = 3$

$y = C_1 e^{3t} + C_2 e^{-2t}$
$+ (4t^2 + 3t)e^{3t} + (2t - 1)$

(c) $\boxed{4}$ $y'' - y' - 6y = 50t\cos t$ [WARNING: There be fractions here matey!]

$y_p = (At+B)\cos t + (Ct+D)\sin t$

$y_p' = (Ct+D+A)\cos t + (C - At - B)\sin t$

$y_p'' = (2C - At - B)\cos t + (-Ct - D - 2A)\sin t$

Substituting gives

$y_p'' - y_p' - 6y_p = [2C - At - B - Ct - D - A - 6At - 6B]\cos t$
$+ [-Ct - D - 2A - C + At + B - 6Ct - 6D]\sin t = 50t\cos t$

$t\cos t$: $-7A - C = 50$ $\cos t$: $2C - A - 7B - D = 0$

$t\sin t$: $A - 7C = 0$ $\sin t$: $B - 2A - C - 7D = 0$

so $A = -7$, $C = -1$ $B = 2/5$, $D = 11/5$

$y = C_1 e^{3t} + C_2 e^{-2t} + \left(-7t + \frac{2}{5}\right)\cos t + \left(-t + \frac{11}{5}\right)\sin t$