Math 274 Homework **Name:** ____________________

Sections: 1.1-1.3, Phase Line Point values in $\boxed{\text{boxes}}$.

Due: May 2018

1. $\boxed{2}$ Verify $\phi(x) = \dfrac{x}{\ln|x| + c}$, where c is an arbitrary constant, is a one-parameter family of solutions to
$$\frac{dy}{dx} = \frac{y(x-y)}{x^2}.$$

$$\varphi'(x) = \frac{\ln|x| + c - x\left(\frac{1}{x}\right)}{(\ln|x| + c)^2}$$

$$= \frac{\ln|x| + c - 1}{(\ln|x| + c)^2}$$

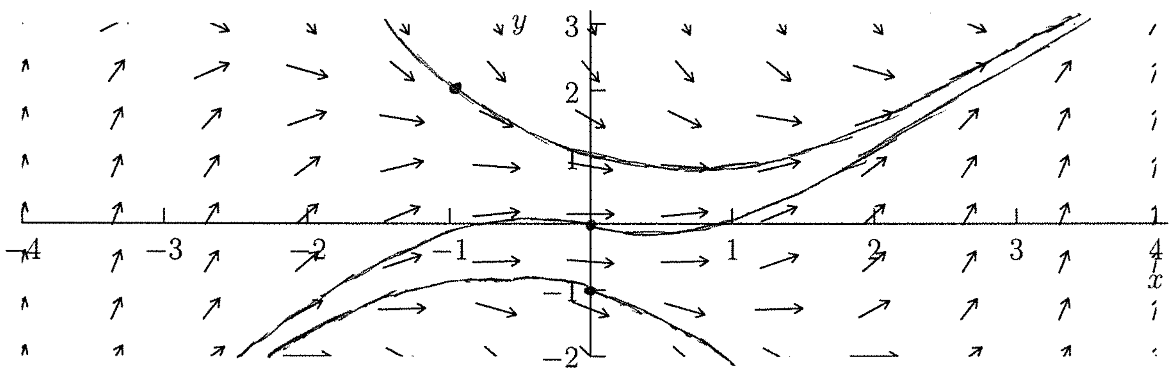
$$\frac{\varphi(x-\varphi)}{x^2} = \frac{1}{x^2}\left(\frac{x}{\ln|x|+c}\right)\left(x - \frac{x}{\ln|x|+c}\right)$$

$$= \frac{1}{\ln|x|+c}\left(\frac{\ln|x|+c-1}{\ln|x|+c}\right)$$

$$= \frac{\ln|x|+c-1}{(\ln|x|+c)}$$

so $\varphi' = \dfrac{\varphi(x-\varphi)}{x^2}$

2. $\boxed{2}$ The direction field for $\frac{dy}{dx} = f(x,y)$ is shown below.



Sketch the solution curves satisfying the following initial data.

(a) $y(0) = 0$ (b) $y(-1) = 2$ (c) $y(0) = -1$

3. $\boxed{2}$ For each of the following equations, label the equilibria and classify each as a sink, source, or node on the phase line. Include direction arrows. Additionally, for each, predict the asymptotic behavior as $t \to \infty$ of the solution satisfying $y(0) = 0.9$.

(a) $y' = y^4 - y^2 = y^2(y^2-1)$

$y=-1$ $y=0$ $y=1$

sink node source

$y \longrightarrow 0$ as $t \to \infty$

(b) $y' = y^4 - y^2 + 0.1$

$y \approx -0.94$ $y \approx -0.34$ $y \approx 0.34$ $y \approx 0.94$

sink source sink source

$y \longrightarrow \approx 0.34$ as $t \to \infty$

4. $\boxed{1}$ Review Integration by Parts and evaluate

$$\int x^2 e^{-3x}\, dx.$$

$u = x^2 \qquad dv = e^{-3x}\,dx$

$du = 2x\,dx \qquad v = -\frac{1}{3}e^{-3x}$

$$= -\frac{1}{3}x^2 e^{-3x} + \int \frac{2}{3} x e^{-3x}\, dx = -\frac{1}{3}x^2 e^{-3x} - \frac{2}{9} x e^{-3x} + \int \frac{2}{9} e^{-3x}\, dx$$

$u = \frac{2}{3}x \qquad dv = e^{-3x}\,dx$

$du = \frac{2}{3}dx \qquad v = -\frac{1}{3}e^{-3x}$

$$= -\frac{1}{3}x^2 e^{-3x} - \frac{2}{9} x e^{-3x} - \frac{2}{27} e^{-3x} + C$$

5. $\boxed{1}$ Review Partial Fraction Decomposition and evaluate

$$\int \frac{2}{y^2-1}\, dy.$$

$$\frac{2}{y^2-1} = \frac{A}{y-1} + \frac{B}{y+1}$$

$2 = A(y+1) + B(y-1)$

$y=1$: $2 = 2A$ so $A=1$

$y=-1$: $2=-2B$ so $B=-1$

$$= \int \left(\frac{1}{y-1} - \frac{1}{y+1} \right) dy$$

$$= \ln|y-1| - \ln|y+1| + C$$

6. $\boxed{1}$ Read section 2.2 in your text. The differential equation

$$\frac{dy}{dx} = \frac{x^2y^2 - x^2}{2e^{3x}} \tag{1}$$

is separable, find an implicit general solution. You should find the integrals above useful.

Seperating gives $\int \frac{2}{y^2-1}\,dy = \int x^2 e^{-3x}\,dx$, by above we have

$$\ln|y-1| - \ln|y+1| = e^{-3x}\left(-\frac{x^2}{3} - \frac{2x}{9} - \frac{2}{27}\right) + C$$

7. $\boxed{1}$ Verify that both $y \equiv 1$ and $y \equiv -1$ are solutions to (1). Both solutions were lost when we solved the equation; when were they lost?

When we separated we lost solutions of the form $y^2 - 1 \equiv 1$, i.e.

$y \equiv 1$ and $y \equiv -1$. Be careful when dividing to make sure it is not zero.